A Novel Evidence Distance in Power Set Space

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Abstract: *Distance measure of evidence presented has been used to measure the similarity of two bodies of evidence. However, it is not considered that the probability distribution on a power set is able to assign to its subsets not only single elements. In this paper a novel approach is proposed to measure the distance of evidence. And some properties that the novel approach has, such as nonnegativity, symmetry, triangular inequality, downward compatibility and higher sensitivity, is proved. Numerical example and real application are used to strictly illustrate the efficiency of the new distance.*

Keywords: *Evidence theory, evidence distance, data function, target recognition system.*

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1. Introduction

Since Dempster-Shafer theory (DS theory) is proposed [4, 21], it has been widely used in information fusion [2, 9, 14, 19, 27, 31], decision making [5, 7, 20, 30] and other fields [1, 3, 6, 8, 13, 18, 28]. However, Zadeh *et al*. [26] proposed a numerical example to illustrate that it may result an illogical result when two bodies of evidences are in conflict. Therefore, how to describe conflict and difference between two bodies of evidence becomes important. The metric distance not only quantitatively describe the difference between two bodies of evidence, but also is used to calculate the weight in weighted average combination rule, which is a broadly accepted method [17, 24].

A lot of works have been done to introduce quantitative measuring method [11, 16]. In [15], the point that the conflict coeffcient in evidence theory is able to represent the conflict between evidences is illustrated. The widely used and classical method is constructing a map from vectors made of Basic Probability Assignment (BPA) to real number presented in [10, 12, 22, 23]. Some real applications indicate the evidence distance can show the difference between two bodies of evidence. What’s more important, the function satisfies three characteristics of distance, namely nonnegativity, symmetry and triangle inequality.

However, this method doesn’t have a fast convergence and it is not considered that probability distribution on a power set is able to assign to its subsets not only single elements. In other words, the similarity between two sets is not just the similarity among single elements, their power sets should be considered as well.

In this paper, a novel distance between two bodies of evidence is proposed based on classical evidence distance. In the novel evidence the first step is to constitute vectors made of BPAs. And then a matrix D_p whose elements represent the similarity between two sets A and B, like metric in Euclid space, is produced. The last step is to calculate the distance using vectors and matrixD_p like calculating the length of difference between two vectors.

The rest of the paper is organized as follows. In section 2, Dempster-Shafer theory and the existing evidence distance are briefly introduced. In section 3, a new evidence distance is proposed, and five properties of new distance function has are proved as well. In section 4 a numerical example is used to illustrate the behaviour of the new evidence distance. We also use it in the real application to exam its practical applicability. Section 5 concludes the main contribution of the paper.

2. Preliminaries

In this section, some preliminaries are briefly introduced below.

2.1. Dempster-Shafer Theory of Evidence

The theory of evidence is introduced by Dempster [4] and developed by Shafer [21]. In Dempster-shafer theory, basic probability is distributed to power sets not mutually exclusive elements. Some terminology and notions are defined below to explain theory better.

Let Θ be a set of N mutually exclusive and exhaustive elements, which means the problem has N possible values. The following set is called the frame of discernment

$$\Theta = \{H_1, H_2, ..., H_N\}. \tag{1}$$

P(Θ) is the power set composed of 2^N elements A of Θ, representing the object is in A

$$P(\Theta) = \{\phi, H_1, \cdots, H_N, (H_1, H_2), (H_1, H_3), \ldots, (H_{N-1}, H_N), \cdots, (H_1, H_2, H_3), \cdots, \Theta\}. \quad (2)$$

A Basic Probability Assignment (BPA) is a function from P(Θ) to [0, 1] defined by:

$$m: \quad P(\Theta) \rightarrow [0,1] \quad (3)$$

and which satisfies the following conditions [4, 21]:

$$\sum_{A \in P(\Theta)} m(A) = 1, \quad (4)$$

$$m(\phi) = 0. \quad (5)$$

Where m(A) represents the belief to A. In other words, m(A) is the support that we only know the object is belong to A, but we don't accurately know which it is. If m(A) =1, it represents it is certain that the object is in A, which means we completely have confidence in this set. If m(A) = 0, it represents that we have no confidence in set A. Two bodies of evidence, $m_1$andm_2, can be combined to yield a new evidence m, by the follow combination rule [4, 21]

$$m(A) = \begin{cases} \dfrac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K} & A \neq \phi \\ 0 & A = \phi \end{cases} \quad (6)$$

with

$$K = \sum_{B \cap C = \phi} m_1(B) m_2(C) \quad (7)$$

Where K is named conflict coefficient. It reflects the conflict between two bodies of evidences. Absolutely,0 ≤*K*≤ 1.*K* = 0 shows the absence of conflict between two bodies of evidence. *K*=1 shows complete conflict between $m_1$and m_2. When *K*=1, the Dempster's rule of combination is no longer applicable.

- *Example* 2.1. Given Θ = {a, b, c}, considering the following two bodies of evidence.

m_1: m_1({a,b})=0.5, m_1(c)=0.5;
m_2: m_2({a,b})=0.5, m_2(c)=0.5.

The two bodies of evidence are completely identical. But conflict coefficient *K* is means the two bodies of evidence have great conflict. It is obvious that using conflict coefficient to measure the distance will cause the counterintuitive result.

In addition, as long as BPA of a body of evidence is distributed to sets with no intersection, K is always greater than 0, even though m_1 and m_2 are completely identical. Therefore K is not able to accurately represent the conflict of two bodies of evidence.

2.2. Existing Evidence Distance

To measure the distance between two bodies of evidence, Joussel me defined a function from vector made up of BPAs to real number. Let m_1 and m_2 be two BPAs on the same frame of discernment Θ, containing *N* mutually exclusive and exhaustive hypotheses. The distance between m_1 and m_2 is [18]:

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\overrightarrow{m_1} - \overrightarrow{m_2})^T \underline{\underline{D}}(\overrightarrow{m_1} - \overrightarrow{m_2})}, \quad (8)$$

Where $\overrightarrow{m_1}$ and $\overrightarrow{m_2}$ are the associated vectors of BPAs $m_1$and m_2 and $\underline{\underline{D}}$ is a $2^N \times 2^N$matrix whose elements are

$$\underline{\underline{D}}(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad (9)$$

$$A, B \in P(\Theta) \quad (10)$$

Equation (8) can be transformed as follows [18]

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}\left(\|\overrightarrow{m_1}\|^2 + \|\overrightarrow{m_1}\|^2 - 2\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle\right)} \quad (11)$$

Where $\|\vec{m}\|^2 = \langle \vec{m}, \vec{m} \rangle$ and $\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle$ are the scalar product defined by

$$\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle = \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \quad (12)$$

with $A_i, A_j \in P(\Theta),\ i, j = 1, 2, \cdots, 2^N$.

This function d_{BPA} (m_1, m_2) satisfies three requirements, namely nonnegativity, symmetry, triangle inequality. We use Equation (8) to calculate the distance between m_1 and m_2 in Example 1, we obtain that the result is zero, which is consistent with real condition.

3. The Proposed Evidence Distance

In this section, the definition and the properties of the proposed evidence distance function are detailed.

3.1. Definition

When calculating evidence distance, a matrix $\underline{\underline{D}}$ is used in [10], whose elements, Jaccard similarity coefficient, are

$$\underline{\underline{D}}(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad A, B \in P(\Theta), \quad (13)$$

to measure distance between two subsets A and B of Θ . But in Dempster-Shafer theory, belief value is distributed to power sets. For example, we set a frame of discernment Θ as follows [10]:

$$\Theta = \{a, b, c, d\}. \quad (14)$$

In addition, two sensors produce two pieces of data,m_1 (A)=0.2, m_2 (B)=0.5. And A = {a, b, c}, B ={c, d} are elements of P(Θ). It illustrates that the first sensor thinks the object possibly is single elements a, b, c or its subsets {a, b},{a, c},{b, c},{a, b, c}. It is notable to distribute belief function on single elements, but the global possibility is 0.2. We can regard A as a new frame of discernment. So the belief of A is divided by$(2^{|A|}-1)$, which represents the potential number of states in A (We exclude the empty set of A). Similarly B is equivalent to a frame of discernment whose global possibility is 0.5. The union set and intersection of A and B are able to regard as new discernments, which will assign basic probability on power sets.

Therefore,

$$D_p(A,B)=\frac{2^{|A\cap B|}-1}{2^{|A\cup B|}-1} \tag{15}$$

Can reflect the similarity between set A and B more reasonably.

Then the definition of novel distance is presented below.

Let m_1 and m_2 be two BPAs on the same frame of discernment Θ, containing N mutually exclusive and exhaustive hypotheses. The distance between m_1 and m_2 is:

$$d_{BPA}(m_1,m_2)=\sqrt{\frac{1}{2}(\overrightarrow{m_1}-\overrightarrow{m_2})^T D_p(\overrightarrow{m_1}-\overrightarrow{m_2})}, \tag{16}$$

Where $\overrightarrow{m_1}$ and $\overrightarrow{m_2}$ are the associated vectors of BPAs $\overrightarrow{m_1}$ and $\overrightarrow{m_2}$, D_p is an $2^N\times 2^N$ matrix whose elements are

$$D_p(A,B)=\frac{2^{|A\cap B|}-1}{2^{|A\cup B|}-1},\quad A,B\in P(\Theta). \tag{17}$$

Similarly, (16) can be transformed as follows

$$d_{BPA}(m_1,m_2)=\sqrt{\frac{1}{2}\left(\|\overrightarrow{m_1}\|^2+\|\overrightarrow{m_2}\|^2-2\langle\overrightarrow{m_1},\overrightarrow{m_2}\rangle\right)} \tag{18}$$

where $\|\vec{m}\|^2=\langle\vec{m},\vec{m}\rangle$, and $\langle\overrightarrow{m_1},\overrightarrow{m_2}\rangle$ are the scalar product defined by

$$\langle\overrightarrow{m_1},\overrightarrow{m_2}\rangle=\sum_{i=1}^{2^N}\sum_{j=1}^{2^N}m_1(A_i)m_2(A_j)\frac{2^{|A_i\cap A_j|}-1}{2^{|A_i\cup A_j|}-1} \tag{19}$$

with $A_i, A_j\in P(\Theta)$, $i,j=1,2,\cdots,2^N$.

3.2. Properties of Novel Evidence Distance

First, the new distance satisfies the following requirements for any vectors made of BPAs.

1. Nonnegativity: $d(m_1,m_2)\geq 0$ if and only if $m_1=m_2$, $d(m_1,m_2)=0$.
2. Symmetry: $d(m_1,m_2)=d(m_2,m_1)$.
3. Triangleinequality $d(m_1,m_3)\leq d(m_1,m_2)+d(m_2,m_3)$.

Next we prove that this function has three properties. Let m_1, m_2 be two BPAs on the same frame of discernment $\Omega(|\Omega|=n)$. Then the distance between them is

$$d_p(m_1,m_2)=\sqrt{\frac{1}{2}(m_1-m_2)^T D_{p_n}(m_1-m_2)}, \tag{20}$$

Where D_{p_n} is an $2^N\times 2^N$ matrix whose elements are

$$ent_{ij}D_{p_n}=\frac{2^{|A_i\cap A_j|}-1}{2^{|A_i\cup A_j|}-1}, A_i, A_j\subset\Omega. \tag{21}$$

We prove nonnegativity the function has. When n =1, $D_{p_1}=\begin{pmatrix}1 & 1/2\\ 1/2 & 1\end{pmatrix}$ is obviously positive definite matrix.

We suppose that matrix D_{p_k} is positive definite, when n= k. Transform Dp_{k+1} to $D'_{p_{k+1}}$ by exchanging two rows or two columns a couple of times. In other words, $D_{p_{k+1}}=P\times D'_{p_{k+1}}\times Q$,where P, Q are invertible matrix. So $D_{p_{k+1}}$ and $D'_{p_{k+1}}$ have the same positive definite property and

$$D_{p_{k+1}'}=\begin{pmatrix}D_{p_k} & \frac{1}{2}D_{p_k}\\ \frac{1}{2}D_{p_k} & D_{p_k}\end{pmatrix}. \tag{22}$$

For any two vectors $x_1, x_2\in R^{2^k\times 1}$ satisfy that

$$\begin{aligned}\begin{pmatrix}x_1\\x_2\end{pmatrix}^T D'_{p_{k+1}'}\begin{pmatrix}x_1\\x_2\end{pmatrix}&=(x_1^T\,x_2^T)D'_{p_{k+1}'}\begin{pmatrix}x_1\\x_2\end{pmatrix}\\&=(x_1^T\,x_2^T)D_{p_{k+1}'}\begin{pmatrix}x_1\\x_2\end{pmatrix}\\&=(x_1^T\,x_2^T)\begin{pmatrix}D_{p_k} & \frac{1}{2}D_{p_k}\\ \frac{1}{2}D_{p_k} & D_{p_k}\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}\\&=x_1^T D_{p_k}x_1+\frac{1}{2}x_2^T D_{p_k}x_1+\frac{1}{2}x_1^T D_{p_k}x_2\\&\quad+x_2^T D_{p_k}x_2\\&=\frac{1}{2}[x_1^T D_{p_k}x_1+x_2^T D_{p_k}x_2+\\&\quad(x_1+x_2)^T D_{p_k}(x_1+x_2)],\end{aligned} \tag{23}$$

Because of the nonnegativity of D_{p_k} ,we can draw a conclusion that

$$x_1^T D_{p_k}x_1+x_2^T D_{p_k}x_2+(x_1+x_2)^T D_{p_k}(x_1+x_2)\geq 0, \tag{24}$$

If and only if $x_1=x_2=0$, the equality holds.

Thus, D_{pk+1} is positive definite. By the mathematical induction, D_{p_n} is positive definite, and $d(m_1,m_2)$ is positive definite. Next we prove the function satisfies Symmetry. We note that

$$\begin{aligned}d(m_2,m_1)&=\sqrt{\frac{1}{2}(\overrightarrow{m_2}-\overrightarrow{m_1})^T D_{p_n}(\overrightarrow{m_2}-\overrightarrow{m_1})}\\&=\sqrt{\frac{1}{2}[-(\overrightarrow{m_1}-\overrightarrow{m_2})]^T D_{p_n}[-(\overrightarrow{m_1}-\overrightarrow{m_2})]}\\&=\sqrt{\frac{1}{2}(\overrightarrow{m_1}-\overrightarrow{m_2})^T D_{p_n}(\overrightarrow{m_1}-\overrightarrow{m_2})}\\&=\mathrm{d}(\mathrm{m}_1,\mathrm{m}_2)\end{aligned} \tag{25}$$

Thus, $d(m_1,m_2)$ satisfies Symmetry.

Then we prove triangle inequality the modified distance has. For D_{p_n} is positive definite,by Cholesky decomposition, we have

$$D_{p_n}=C^T C, \tag{26}$$

Where $C\in R^{2^n\times 2^n}$ is an invertible matrix, so we can obtain the following equation

$$
\begin{aligned}
d(m_1,m_2) &= \sqrt{\frac{1}{2}(\vec{m_1}-\vec{m_2})^T C^T C(\vec{m_1}-\vec{m_2})}. \\
&= \sqrt{\frac{1}{2}(C(\vec{m_1}-\vec{m_2}))^T (C(\vec{m_1}-\vec{m_2}))} \\
&= \frac{\sqrt{2}}{2} \| C(m_1-m_2) \|_2
\end{aligned} \tag{27}
$$

For 2-norm satisfies Triangle inequality, we can draw a conclusion that also satisfies Triangle inequality.

In addition, when the belief value is only assigned to single elements, the modified matrix D_p degenerates to matrix D.

$$
\frac{2^{|A\cap B|}-1}{2^{|A\cup B|}-1} = \frac{|A\cap B|}{|A\cup B|} \tag{28}
$$

In other words, when the belief value is only assigned to single elements, the proposed method is the same as the method represented by Jousselme.

- *Example* 3.1. Set a frame of discernment Ω = {a, b, c},and two BPAs are given as follows

m_1: $m_1(a)=0.6$, $m_1(b)=0.4$,
m_2: $m_2(a)=0.6$, $m_2(c)=0.4$

According to Equations (16) and (17), we obtain $d_{BPA}= d_{BPA} = 0.32$.

Compare with classical evidence distance, the modified method converges faster and has higher sensitivity. The proof is given below.

We suppose that |A ∪ B| = a, |A ∩ B| = b, x = a− b, Thus

$$
\begin{aligned}
\underline{\underline{D}}(A,B) &= \frac{a-x}{a}, \\
D_p(A,B) &= \frac{2^{a-x}-1}{2^a-1}.
\end{aligned} \tag{29}
$$

Then we obtain that the derivative of $\underline{\underline{D}}(A, B)$ and D(A, B) are

$$
\begin{aligned}
\frac{d(\underline{\underline{D}}(A,B))}{dx} &= -\frac{1}{a} \\
\frac{d(D_p(A,B))}{dx} &= -\frac{2^a-1}{\ln 2\cdot 2^{a-x}}.
\end{aligned} \tag{30}
$$

We find that D_p (A, B) decreases quickly with the increase of *x*. So we think the new evidence distance converges faster and has a higher sensitivity than evidence distance in [10]. Example 3 specifically shows this property.

4. Numerical Example and Real Application

In this section numerical example and real application exams are used to illustrate the behavior and practical applicability of the new evidence distance.

4.1. Numerical Example

An example is illustrated to show the advantage of the new method.

- *Example* 4.1.Set a frame of discernment Ω = {1, 2, 3,…, 20}, and two BPAs are given as follows
  m_1: $m_1(7) = 0.6$, $m_1(A) = 0.4$,

  m_2: $m_2(1, 2, 3) = 1$, with A goes through {1}, {1, 2}, {1, 2, 3},…, {1, 2, 3, …, 10}.

4.2. Real Application

Evidence distance is widely used in target recognition system [25].

The results by different methods to measure the distance between two bodies of evidence are shown in Figure 1 and Table 1.

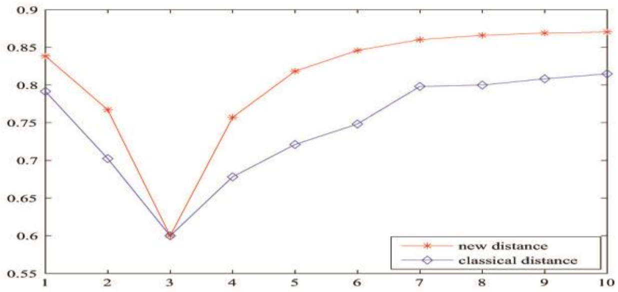

Figure 1. Comparison between two kinds of evidence distance.

As can be seen in Figure 1, with two bodies of evidence approaching, the new evidence distance will decrease. And when A={1, 2, 3}, the new evidence distance achieves the minimum. With the gap between two bodies of evidence increasing, the new distance increases as well. This new evidence distance curve can quantitatively reflect the difference between two bodies of evidence.

Compared with evidence distance curve, the new evidence distance curve varies quickly when A near the set {1, 2, 3}. As A varies from {1, 2} to {1, 2, 3} the new distance decreases 0.1672, while the classical distance decreases 0.1024. As A varies from {1, 2, 3} to {1, 2, 3, 4} the new distance increases 0.1572, while the classical distance increases 0.0782. The new evidence distance can obviously show the nearest point of two bodies of evidence.

When A is set {1, 2, 3}, the matrix $\underline{\underline{D}}$ is the same an D_P, two kinds of distance have the same value, since the novel function degenerates to the classical evidence distance.

From Figure 1 we can conclude that the modified evidence distance has a faster convergence speed and a higher sensitivity.

Table 1. Comparison between two kinds of evidence distance.

A	Classical evidence distance	Modified evidence distance
{1}	0.7916	0.8383
{1, 2}	0.7024	0.7672
{1, 2, 3}	0.6000	0.6000
{1, 2, 3, 4}	0.6782	0.7572
{1, 2, 3, 4, 5}	0.7211	0.8183
{1, 2, 3, 4, 5, 6}	0.7483	0.8459
{1, 2, 3, 4, 5, 6, 7}	0.7982	0.8601
{1, 2, 3, 4, 5, 6, 7, 8}	0.8000	0.8660
{1, 2, 3, 4, 5, 6, 7, 8, 9}	0.8083	0.8689
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}	0.8149	0.8703

- *Example* 4.2. Another application from reference [29] is illustrated to demonstrate the effectiveness of the proposed method. Assume that there are three objects A, B, C, in a target recognition system. The frame of discernment is denoted by Θ ={A, B, C}.In the WSN, there are five difference kinds of sensors observing objects which are CCD sensor(S1), sound sensor(S2), infrared sensor(S3), radar sensor(S4) and ESM sensor(S5). The evidences obtained from these five kinds of sensors are shown in Table 2.

Table 2. Five evidence obtained by sensors.

	{A}	{B}	{C}	{A,C}
S 1:m1(·)	0.41	0.29	0.3	0
S 2:m2(·)	0	0.9	0.1	0
S 3:m3(·)	0.58	0.07	0	0.35
S 4:m4(·)	0.55	0.1	0	0.35
S 5:m5(·)	0.6	0.1	0	0.3

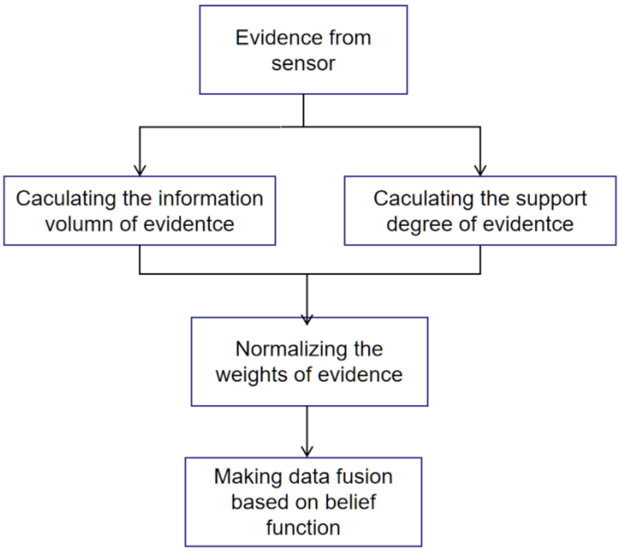

Figure 2. The flowchart of the new method.

It is obvious that the second evidence is abnormal. It will lead to a counterintuitive result after fusion.

We use the new evidence distance replacing classical distance and the combination rule presented in [25] to deal with these evidence. The specific flowchart of the method that we use is shown in Figure 2. And results are shown in Table 3 and Figure 3 compared with different combination rules. When five

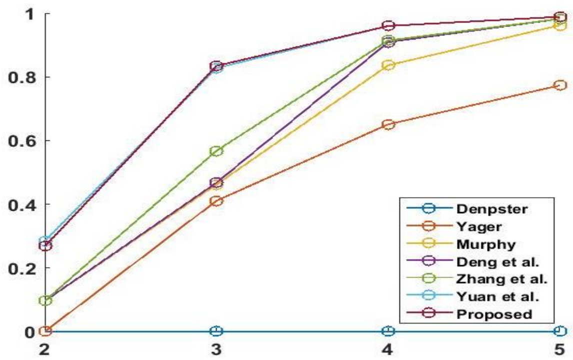

Figure 3. The fusion result comparison of different rules.

bodies of evidence are obtained and the proposed method is used, the calculation process is given below.

First, adopt Equation (17) to calculate the distance and then calculate the support degree of evidences.

Sup(1) = 3.4222
Sup(2) = 2.1219
Sup(3) = 3.7734
Sup(4) = 3.8213
Sup(5) = 3.8057

Next, obtain the information value Iv(i) $(1 \leq i \leq 5)$of each evidence.

Iv(1) = 4.7893
Iv(2) = 1.5984
Iv(3) = 6.1056
Iv(4) = 6.6287
Iv(5) = 5.8764

Then, obtain the weight of each evidence.

w1= 0.1811
w2= 0.0375
w3= 0.2545
w4= 0.2798
w5= 0.2471

Finally, modify the BPAs by weights and combine the weighted averaging evidence four times. The final results are listed in Table 3.

Though the second evidence is completely different with others, which will make distribution when making decision. When it comes to three kinds of evidence, according to calculating the distance among sensor 1, sensor 2, and sensor 3, similarity results are given greater weight in modified method, the belief degree on A reaches up to 0.8345. When it comes to five kinds of evidence, the belief degree on A reaches up to 0.9887, for which we can make decision certainly. It is not distributed by the second evidence. Comparing with other methods shown above, the modified method assign A higher belief value which is not distributed by the wrong evidence. So the modified method using new evidence distance is an efficient method in dealing with conflict.

Table 3. Fusion result.

Combination rule	$\{m_1, m_2\}$	$\{m_1, m_2, m_3\}$	$\{m_1, m_2, m_3, m_4\}$	$\{m_1, m_2, m_3, m_4, m_5\}$
Dempster	m(A) = 0	m(A) = 0	m(A) = 0	m(A) = 0
	m(B) = 0.8969	m(B) = 0.6575	m(B) = 0.3321	m(B) = 0.1422
	m(C) = 0.1031	m(C) = 0.3425	m(C) = 0.6679	m(C) = 0.8578
Yager	m(A) = 0	m(A) = 0.4112	m(A) = 0.6508	m(A) = 0.7732
	m(B) = 0.260	m(B) = 0.0679	m(B) = 0.0330	m(B) = 0.0167
	m(C) = 0.0300	m(C) = 0.0105	m(C) = 0.0037	m(C) = 0.0011
	m(AC) = 0	m(AC) = 0.2481	m(AC) = 0.1786	m(AC) = 0.0938
Murphy	m(A) = 0.0964	m(A) = 0.4619	m(A) = 0.8362	m(A) = 0.9620
	m(B) = 0.8118	m(B) = 0.4497	m(B) = 0.1147	m(B) = 0.0210
	m(C) = 0.0917	m(C) = 0.0794	m(C) = 0.0410	m(C) = 0.0138
	m(AC) = 0	m(AC) = 0.0090	m(AC) = 0.0081	m(AC) = 0.0032
Deng *et al.*	m(A) = 0.0964	m(A) = 0.4674	m(A) = 0.9089	m(A) = 0.9820
	m(B) = 0.8119	m(B) = 0.4054	m(B) = 0.0444	m(B) = 0.00039
	m(C) = 0.0917	m(C) = 0.0888	m(C) = 0.0379	m(C) = 0.0107
	m(AC) = 0	m(AC) = 0.0084	m(AC) = 0.0089	m(AC) = 0.0034
Zhang *et al.*	m(A) = 0.0964	m(A) = 0.5681	m(A) = 0.9142	m(A) = 0.9820
	m(B) = 0.8119	m(B) = 0.3319	m(B) = 0.0395	m(B) = 0.00034
	m(C) = 0.0917	m(C) = 0.0929	m(C) = 0.0399	m(C) = 0.0115
	m(AC) = 0	m(AC) = 0.0084	m(AC) = 0.0083	m(AC) = 0.0032
Yuan *et al.*	m(A) = 0.2849	m(A) = 0.8274	m(A) = 0.9596	m(A) = 0.9886
	m(B) = 0.5306	m(B) = 0.0609	m(B) = 0.0032	m(B) = 0.00002
	m(C) = 0.1845	m(C) = 0.0986	m(C) = 0.0267	m(C) = 0.0072
	m(AC) = 0	m(AC) = 0.0131	m(AC) = 0.0106	m(AC) = 0.0039
Proposed method	m(A) = 0.2678	m(A) = 0.8345	m(A) = 0.9598	m(A) = 0.9887
	m(B) = 0.5552	m(B) = 0.0622	m(B) = 0.0039	m(B) = 0.0003
	m(C) = 0.1770	m(C) = 0.0865	m(C) = 0.0250	m(C) = 0.0070
	m(AC) = 0	m(AC) = 0.0167	m(AC) = 0.0114	m(AC) = 0.0040

5. Conclusions

How to measure dissimilarity between two bodies of evidence is a significant problem. If conflict coefficient *K* is used to reflect the difference between two bodies of evidence, it will produce counterintuitive conclusion, especially when the two bodies of evidence have a great difference. In evidence theory, belief value is assigned to power set but not singleton. So any subsets of discernment including m elements can produce a 2^m-1 linear space. The new evidence distance base on this thought improves evidence distance. Let $D_p(A, B)$ express similarity between A and B, so that the distance matrix D whose element is D(A, B) is more meaningful and accurate as metric matrix. By reasoning and several examples, we know that novel evidence distance preserves properties and advantage of the evidence distance introduced by Jousselme *et al.* [10] In addition, when the belief value is only assigned to single elements the novel function degenerates to the classical evidence distance. Compared with evidence distance introduced by Jousselme *et al.* [10] new evidence distance we propose in this paper has a higher sensitivity, which can accurately represent conflict of two bodies of evidence, as mentioned above.

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