# Genetic Algorithm-Based Synthesis of Three-Dimensional Microstrip Arrays 

Sidi Ahmed Djennas, Fethi Bendimerad, Lotfi Merad, and Sidi Mohamed Meriah<br>Laboratory of Telecommunications, Abou-Bekr Belkaid University-Tlemcen, Algeria


#### Abstract

The radiation pattern synthesis of rectangular microstrip patches arranged on three-dimensional arrays using numerical deterministic optimization methods was vulnerable to the local optima problem. Then, the use of probabilistic optimization methods constitutes an attractive alternative. Hence, these methods give the opportunity to deviate from the local optima to global optima. The local aspect of the findings is a frequent problem for the calculus-based methods, where the obtained solution depends on the starting point. It is towards this perspective that we develop in this paper a synthesis based on genetic algorithm which remains a probabilistic optimization method, easy to integrate in computations and very efficient even though it use probabilistic and deterministic rules, particularity which makes the efficiency and robustness of this heuristic algorithm.


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## 1. Introduction

Since approximately two decades the civilian and military aerospace applications, e. g., (control of air traffic, communications links, radio guidance, telemetry, satellites), needs Omni-directional coverage with a new type of arrays $[1,2,3,4,9,13,14]$.

These arrays have a three-dimensional configuration which characterizes them from the other types of rectilinear or planar arrays. They are more appropriate for the communication links, but their modeling remains a very complex problem, because each elementary pattern points a different direction in space. In this case, the computation of these elementary patterns is essential, before getting to their weighted summation and finally finding the values of the farfield pattern [14].

This process is applied to the radiation analysis. By the same way, this formalism is undertaken for the synthesis which foresee in our paper, the finding of the optimal feed vector according to specified destination pattern.

Numerical deterministic optimization methods can solve this problem, but remains limited, regarding the local optima difficulties. The alternative would be then to use the probabilistic optimization methods which are not limited by restrictive assumptions about the search space $[5,6,7,8,10,11,12,15]$.

## 2. Problem Formulation and Genetic Algorithm

The genetic algorithm is inspired from the biology and the natural selection laws. This selection guarantees
perenniality to the fittest individual and to his descendants that will take part in a future procreation cycle. Thus, at the end of several generations, we always preserve the fittest individual [5, 7, 11, 15]. This selective aspect will constitute the link between the genetic algorithm and any optimization problem. In our case, the fittest individual is the optimal feed vector of the three-dimensional microstrip array. During a generation, three types of genetic operators are applied to the individuals, i. e., cross-over, mutation, selection and evaluation [5].

The far-field pattern of a $\mathrm{N}_{\mathrm{c}}$-elements array is given by the expression (1) as shown in Figure 1.

$$
\begin{equation*}
g(\theta, \varphi)=\sum_{n=1}^{n=N_{c}} s_{n} e^{j \frac{2 \pi}{\lambda}\left(\overrightarrow{\mathrm{om}_{n}} \vec{r}\right)} \psi_{n}(\theta, \varphi) \tag{1}
\end{equation*}
$$

Where:
$\mathrm{S}_{\mathrm{n}}$ : nth element complex feed.
$\lambda$ : Wavelength.
$\overrightarrow{\mathrm{om}_{\mathrm{n}}}$ : nth element position vector.
$\overrightarrow{\mathrm{r}}$ : Propagation direction vector, specified by $\theta$ and $\varphi$.
$\psi_{\mathrm{n}}(\theta, \varphi)$ : nth elementary pattern in the propagation direction.


Figure 1. Element in the global cartesian co-ordinates system.
The desired (destination) pattern is specified by a twe bound function $[1,11]$, such as:

$$
\begin{equation*}
M_{l}(\theta, \varphi) \leq g(\theta, \varphi) \leq M_{u}(\theta, \varphi) \tag{2}
\end{equation*}
$$

$\mathrm{M}_{1}(\theta, \varphi)$ : Lower bound, $\mathrm{M}_{\mathrm{u}}(\theta, \varphi)$ : Upper bound.
The synthesized pattern must be trapped between the two bounds which are represented on Figure 2 for a revolution view and on Figure 3 for a projected view, where the shaded area is the permitted area for the calculated pattern.


Figure 2. Destination pattern in revolution view.


Figure 3. $\varphi_{c}$ projection of Figure 2.
The destination pattern is defined by the following parameters:

- The main-lobe major peak fixed at 0 dB .
- The maximum ripple value in decibels: MRV.
- The maximum side-lobe level in decibels: MSL.
- The major beamwidth in degrees: MXW.
- The minor beamwidth in degrees: MNW.
- The main-lobe scan angle in elevation: $\theta_{\mathrm{c}}$.
- The principal plane in degrees: $\varphi_{\mathrm{c}}$.
- The coverage region in degrees: CR.

For solving the problem, we will build a matrix from the array structure and the space samples in azimuth and in elevation. This matrix which we symbolized by $\Pi$, is formulated by the synoptic diagram on Figure 4 and expression 3. It will be used as a database for the optimization process by the genetic algorithm. The multiplication of $\Pi$ by the feed vector that we will obtain after optimization, will generate the synthesized pattern values.


Figure 4. Synoptic diagram of $\Pi$ matrix construction.

$$
\begin{equation*}
\Pi(\mathrm{k}, 1)=\mathrm{e}^{\mathrm{j} \frac{2 \pi}{\lambda}\left(\overrightarrow{\mathrm{om}_{1}} \overrightarrow{\mathrm{r}_{\mathrm{k}}}\right)} \psi_{1}\left(\theta_{\mathrm{k}}, \varphi_{\mathrm{k}}\right) \tag{3}
\end{equation*}
$$

1: Element index, it varies from 1 to $\mathrm{N}_{\mathrm{c}}$. k : Sample point index, it varies from 1 to $\mathrm{N}_{\mathrm{s}}$. with:

$$
\mathrm{N}_{\mathrm{s}}=\mathrm{N}_{\mathrm{t}} \mathrm{~N}_{\mathrm{p}}, \mathrm{~N}_{\mathrm{t}}=\frac{180^{\circ}}{\delta \theta}+1, \mathrm{~N}_{\mathrm{p}}=\frac{360^{\circ}}{\delta \varphi}+1
$$

$\delta \theta, \delta \varphi$ : Sample points spacing in degrees, respectively in elevation and in azimuth.

## 3. Statement of the Synthesis Procedure

The flowchart of the synthesis based on the genetic algorithm is represented on Figure 5. The achievement of solution is carried out by iterative process which will end when a maximal number of iterations is reached. Indeed, this allows to control the computing time which remains relatively long compared with that required by the calculus-based methods $[10,11]$.

The solution process begins with the introduction of a random population of individuals encoded into binary and consequently, a first evaluation of the optimal feed matrix of the array. The cross-over and mutation will be next, applied to the rows of this matrix. This, will give rise to a new matrix where the genetic codes of the individuals will be decoded and hence to carry out the pattern analysis. The technique consists in computing the pattern values for each
individual by multiplying each row of the matrix in decimal scale by $П$.

The calculation of the pattern, will enable us thereafter to evaluate the fitness function which must be maximal to minimize the variation between the synthesized pattern and the destination pattern. Then, the individuals are sorted out according to a decreasing fitness. Next, we extract the fittest individual which constitutes the new population matrix in the following cycle. This process will be repeated until reach the maximal number of iteration. The last fittest individual constitutes the optimal feed vector.


Figure 5. Genetic algorithm-based synthesis flowchart.
After discretization of the problem, the fitness function, symbolized by Fss, is given by the expression 4:

$$
\begin{equation*}
F s s=1-\frac{\sum_{j=1}^{j=N_{p}} \sum_{i=1}^{i=N_{t}} \alpha(i, j) \cdot\left(B^{-}(i, j)\right)^{\beta(i, j)}}{\sum_{j=1}^{j=N_{p}} \sum_{i=1}^{i=N_{t}} \alpha(i, j) \cdot\left(B_{+}(i, j)\right)^{\beta(i, j)}} \tag{4}
\end{equation*}
$$

Where:
$B_{+}^{-}(i, j)= \begin{cases}g(i, j) \mp M_{u}(i, j) & \text { if } g(i, j)>M_{u}(i, j) \\ 0 & \text { if } M_{u}(i, j)>g(i, j)>M_{1}(i, j) \\ M_{1}(i, j) \mp g(i, j) & \text { if } M_{1}(i, j)>g(i, j)\end{cases}$
$\alpha$ : Linear weight coefficients.
$\beta$ : Power weight coefficients.
$N_{t}, N_{p}, M_{u}, M_{1}, g$ : Have been defined before.
The minus sign in the left-hand side of equation (5) is assigned to the minus sign in the right-hand side. Idem for the plus sign.

The population matrix is identified to the binary code string of the fittest individual, produced on rows of this matrix as much as it is required to obtain a diversified genetic patrimony and consequently a good convergence neither precocious nor stagnant.

### 3.1. Illustrative Example

The genetic code of the fittest individual for a 2 elements array whose feed is coded on 4 bits, two for the amplitude and two for the phase is represented by the string: 10001010 .

- 10: Complex feed amplitude in binary code, $1^{\text {st }}$ element.
- 00: Complex feed phase in binary code, $1^{\text {st }}$ element
- 10: Complex feed amplitude in binary code, $2^{\text {nd }}$ element.
- 10: Complex feed phase in binary code, $2^{\text {nd }}$ element.

The population matrix is then: $\left(\begin{array}{l}10001010 \\ 10001010 \\ 10001010\end{array}\right)$
Here the string is produced three times upon three rows in the matrix.

Specifically to the given example, Figure 6 illustrates the complete cycle of genetic operators applied to the population matrix.

## 4. Results and Discussions

### 4.1. First Results

### 4.1.1. Rooftop Array

The first studied array is the rooftop array, with a tip half angle of $60^{\circ}$ (Figure 7). The frequency of operation is 5 GHz . The inter-element spacing is $0.5 \lambda$. The successive altitudes of the elements, 1 and 8,2 and 7,3 and 6,4 and 5 are respectively: $1.5 \mathrm{~cm}, 3 \mathrm{~cm}, 4.5$ cm , and 6 cm .

The destination pattern is restricted by: MRV $=4$ $\mathrm{dB}, \mathrm{MSL}=-25 \mathrm{~dB}, \mathrm{MNW}=36^{\circ}, \mathrm{MXW}=70^{\circ}, \mathrm{CR}=$ $360^{\circ}$.

The synthesis results obtained for the rooftop array are depicted on Figure 8 and Figure 9 for a non-scan
case. The evolution of the SideLobe Level (SLL) versus iteration number is then represented on Figure 10.


Figure 6. Block diagram of the genetic algorithm cycle.


Figure 7. Rooftop array.


Figure 8. 3-D plot of the synthesized pattern of the rooftop array, non-scan case, $\theta_{\mathrm{c}}=0^{\circ}, \varphi_{\mathrm{c}}=90^{\circ}$.


Figure $9.90^{\circ}$ plane cut of Figure 8.


Figure 10. Plot of SLL versus iteration number.

### 4.1.2. Truncate-ConeA rray

Figure 11 represents the truncate-cone array with 36 elements at the working frequency of 5 GHz . The array has: 6 rings by 6 elements structure.

The radii of the six rings: $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4, \mathrm{C} 5, \mathrm{C} 6$, from the vertex towards the base are respectively equal to: $9.54 \mathrm{~cm}, 12.14 \mathrm{~cm}, 14.74 \mathrm{~cm}, 17.34 \mathrm{~cm}, 19.94 \mathrm{~cm}$, 22.53 cm . The angular spacing between two adjacent elements in circumference is of $18^{\circ}$.


Figure 11. Truncate-cone array.

The imposed parameters on the destination pattern are: $\mathrm{MRV}=5 \mathrm{~dB}, \mathrm{MSL}=-20 \mathrm{~dB}, \mathrm{MNW}=18^{\circ}, \mathrm{MXW}$ $=55^{\circ}, \mathrm{CR}=180^{\circ}$.

The results of the synthesis are depicted on Figure 12 and Figure 13 for the scan case. Figure 14 shows the evolution of the side lobe level versus the iteration number or cycle.


Figure 12. 3-D plot of the synthesized pattern of the truncated-cone array, scan case, $\theta_{\mathrm{c}}=30^{\circ}, \varphi_{\mathrm{c}}=45^{\circ}$.


Figure 13. $45^{\circ}$ plane cut of Figure 12.


Figure 14. Plot of SLL versus iteration number.

### 4.1.3. Comments and Comparison

A similar synthesis based on simulated annealing technique was carried out respectively for the rooftop and truncate-cone arrays with respect to the same destination pattern [4]. This synthesis had generated a radiation pattern with -25 dB side-lobe level for the rooftop array, whereas our result remains close to, with -24 dB side-lobe level. For the truncate-cone array, the synthesis based on the simulated annealing had generated a radiation pattern trapped between the bounds of the destination pattern with -23 dB sidelobe level [4]. Our finding shows a better approach, with -31 dB .
These two cases application, points out the aptitude of the genetic algorithm to solve with better approach and effectiveness the three-dimensional array synthesis problem.

### 4.2. Second Results

### 4.2.1. Paraboloid Array

A first proposed array is a paraboloid-shaped one. This structure can be conformed on the nose of an aircraft fuselage. As it is shown on Figure 15, this array contains 45 elements that operate at the frequency of 3 GHz.
The elements are distributed symmetrically on five rings with decreasing numbers from the base towards the vertex.
The inter-element spacing varies between $0.55 \lambda$ and $0.65 \lambda$. The rings $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4, \mathrm{C} 5$ whose radii are equal to respectively: $12.5 \mathrm{~cm}, 10.9 \mathrm{~cm}, 9.01 \mathrm{~cm}$, $7.07 \mathrm{~cm}, 4.33 \mathrm{~cm}$, and whose altitudes are equal to : 0 $\mathrm{cm}, 6 \mathrm{~cm}, 12 \mathrm{~cm}, 17 \mathrm{~cm}, 22 \mathrm{~cm}$, contains respectively: $13,11,9,7,5$ elements. The vertex-base distance is of 25 cm .


Figure 15. Paraboloid array.

The destination pattern is defined as follow: $\mathrm{MRV}=$ $4 \mathrm{~dB}, \mathrm{MSL}=-25 \mathrm{~dB}, \mathrm{MNW}=40^{\circ}, \mathrm{MXW}=80^{\circ}, \mathrm{CR}=$ $360^{\circ}$.

On Figure 16 and Figure 17 are plotted the results of the synthesis for the non-scan case and on Figure 19
and Figure 20 the results for the scan case. Figure 18 and Figure 21 shows respectively, the evolution of the SLL versus iteration number for the non-scan and scan cases.


Figure 16. 3-D plot of the synthesized pattern of the paraboloid array, non-scan case, $\theta_{c}=0^{\circ}, \varphi_{c}=0^{\circ}$.


Figure 17. 0 plane cut of Figure 16.


Figure 18. Plot of SLL versus iteration number.


Figure 19. 3-D plot of the synthesized pattern of the paraboloid array, scan case, $\theta_{\mathrm{c}}=75^{\circ}, \varphi_{\mathrm{c}}=120^{\circ}$.


Figure 20. 120 plane cut of Figure 19.


Figure 21. Plot of SLL versus iteration number.
These cases of synthesis using the genetic algorithm show well that the parameters of the destination pattern cannot be entirely respected. Whereas, the main-lobe is well trapped in the area delimited by the two bounds, the sidelobe level overshoots the -25 dB to reach -20 dB on Figure 17.

### 4.2.2. Wing Array

The outline of this array is shown on Figure 22. This array can be conformed on an airfoil and is composed of a cylinder section and plane section which together are composed of 50 elements at the working frequency of 5 GHz . There are 20 elements on the cylinder section sub-array which cover $45^{\circ}$ on a cylinder of radius: $\mathrm{r}=16 \mathrm{~cm}$. The plane section sub-array which is $15^{\circ}$ inclined from the horizontal, contains 30 elements on the five sixths of its area.

The structure is 16.8 cm wide on X -axis. The plane section sub-array is 18 cm long. The inter-element spacing is of 4.2 cm on the cylinder section and 3 cm on the plane section.


Figure 22. Wing array.
For the wing array, the destination pattern is restricted by the following parameters: $\mathrm{MRV}=5 \mathrm{~dB}$, $\mathrm{MSL}=-30 \mathrm{~dB}, \mathrm{MNW}=50^{\circ}, \mathrm{MXW}=70^{\circ}, \mathrm{CR}=360^{\circ}$.

The graphical results of the synthesis are plotted on Figure 23 and Figure 24 for the non-scan case and on Figure 26 and Figure 27 for the scan case. Figure 25 and Figure 28 show the SLL evolution versus iteration number.


Figure 23. 3-D plot of the synthesized pattern of the wing array, non-scan case, $\theta_{\mathrm{c}}=0^{\circ}, \varphi_{\mathrm{c}}=90^{\circ}$.


Figure 24. $90^{\circ}$ plane cut of Figure 23.


Figure 25. Plot of SLL versus iteration number.


Figure 26. 3-D plot of the synthesized pattern of the wing array, scan case, $\theta_{\mathrm{c}}=-45^{\circ}, \varphi_{\mathrm{c}}=70^{\circ}$.


Figure 27. $72^{\circ}$ plane cut of Figure 26.


Figure 28. Plot of SLL versus iteration number.
For the wing array, Figure 24 depicts -27 dB sidelobe level, whereas on Figure 27, the sidelobe level reaches -25 dB . Nevertheless, this is a good result, considering the agreement between the destination and the synthesized patterns.

## 5. Conclusion

The genetic algorithm, through the found results, gives the evidence of efficiency for three-dimensional arrays synthesis. Its implementation on computer made it possible to generate optimal solutions.

The four cases of arrays that we treated, using a synthesis technique based on the genetic algorithm, substantiate that the application of such a heuristic algorithm achieved the goals of a most rigorous and global approach towards the best solutions. Such solutions remain difficult to achieve using calculusbased deterministic methods which are too rigid and limited in search space by the local optima difficulties. Moreover, this algorithm is free from all restrictions associated to the integral calculus, derivatives, matrix algebra, discontinuities, etc... Nevertheless, the
required computing time, remains relatively long compared with the deterministic methods but it is there the price to be paid for a better approach.

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Sidi Ahmed Djennas received the State engineer and Magister diplomas in 1997 and 2001, respectively, from Abou Bekr Belkaid University, Tlemcen, Algeria. Currently, he is a junior lecturer in the same university. Also, he is a junior researcher within the Telecommunications Laboratory. He worked on design and development of antenna analysis and synthesis for communication and radar systems.


Fethi Bendimerad received the State engineer diploma in 1983 from Oran Sciences and Technologies University, Algeria, and the Doctorate degree from Nice Sophia Antiplois University, France, in 1989. Currently, he is working as a senior lecturer in Tlemcen University and is serving as the director of the Telecommunications Laboratory. His main area of research is the microwave techniques and radiation, and he is responsible of the antenna section.


Lotfi Merad received the State engineer and Magister diplomas in 1998 and 2001, respectively, from Abou Bekr Belkaid University, Tlemcen, Algeria. He is a junior researcher within the Telecommunications Laboratory. His field of research is centered on the optimization process and their application on antenna array synthesis.


Sidi Mohamed Meriah received the State engineer diploma in 1992 from Polytechnical Normal School, Algeria, and the Magister diploma in 1997 from Abou Bekr Belkaid University, Tlemcen, Algeria. He is a junior lecturer in Tlemcen University and researcher within the Telecommunications Laboratory. He worked on smart array antenna.

