A New OTIS-Arrangment Interconnection Network

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Abstract: *In this paper, we propose a new interconnection network called Optical Transpose Interconnection Systems (OTIS) Arrangement Network which is constructed from the cross product of the factor Arrangement network. This paper utilizes the features of optical transpose interconnection systems networks which use both of electronic and optical networks. Many recent studies have showed that optical transpose interconnection systems is one of the most promising candidates' networks as future high-speed parallel computers. We also introduce general study on the topological properties for the optical transpose interconnection systems -arrangement by obtaining the main topological properties including size, degree, diameter and number of links and then we propose an efficient routing algorithm. The proposed routing algorithm and the derived properties can be used in general for all optical transpose interconnection systems -network, which will save the researchers effort to work on each on the optical transpose interconnection systems -networks individually. This study provides new means for further testing the possibility of the optical transpose interconnection systems _arrangement as alternative parallel computer network.*

Keywords: *Parallel and distributed systems, interconnection networks, arrangement network, OTIS-Networks, topological properties, and routing algorithm.*

Received February 28, 2009; accepted November 5, 2009

1. Introduction

Through the past years, a widely studied interconnection network topology called arrangement graph which was proposed by Day have been presented in the literature [9]. This network has been shown as an attractive alternative to star network which is proposed by Akers [2]. The arrangement network is node and edge symmetric and furthermore it is strongly hierarchical structure. The arrangement graph An, k is regular of degree k(n-k), where n and k be two integers satisfying 1<= k<=n-1, furthermore let <n> = {1,2,….,n} and <k>= {1,2,…,k}. The number of nodes is n!/(n-k)!, and diameter $\lfloor 3/2\ k \rfloor$. For similar number of nodes, the arrangement network has attractive choices than the star graph which has a major problem where it's number of nodes: n! is spread widely over the set of integers as an example, the smallest star graph with 8K nodes is a 40K nodes. So the arrangement graph is an alternative to star graph since the size for the arrangement network n!/(n-k)! allow more flexibility in choosing the number of nodes in the network than the n! for the number of networks in n-star [9].

Optoelectronic and optical networking will become the key enabling technologies of the future communications infrastructure through the elimination of the difficult limitation of bandwidth and bit-error rate which inherent in traditional electromagnetic signal- based communications [1, 12, 16]. Electromagnetic signals carried over copper (or coaxial) wires suffer from loss of strength and are subject to errors due to noise and hence such systems have limited data rates [5]. When copper or coax is replaced by fiber technology the achievable bandwidth is in excess of 50 terabits/second with an almost zero bit-error rate [5]. The full implications of essentially huge bandwidth and extremely low loss rates are only beginning to be recognized and will radically reshape the future network technologies. While in the past the communication link was the bottleneck, this link now holds the potential to become the enabler of new modes of computing far beyond those existing today [5].

Marsden *et al.* were the first to propose the Optical Transpose Interconnection Systems (OTIS) [12]. A number of computer architectures have subsequently been proposed in which the OTIS were used to connect different processors [12]. Krishnamoorthy *et al.* [11] have shown that the power consumption is minimized and the bandwidth rate maximized when the OTIS computer is partitioned into N groups of N processors each. Zane *et al* [22] have limited their study to this type of the OTIS. In this paper, we focused on OTIS-Networks where the number of processors in each group is equal to the number of groups; the terms OTIS-Computer and OTIS-Network refer to parallel architectures based on the OTIS and will be used interchangeably.

OTIS-networks are implemented using free-space optoelectronic technology [4, 12, 20]. In this model, processors are partitioned into groups, where each group is realized on a separate chip with electronic inter-processor connects. Processors on separate chips are interconnected through free space interconnects. The philosophy behind this separation is to utilise the benefits of both the optical and electronic technologies [14, 15].

The advantage of using the OTIS as optoelectronic architecture lies in its ability to manoeuvre the fact that free space optical communication is superior in terms of speed and power consumption when the connection distance is more than few millimetres [11]. In the OTIS, shorter (intra-chip) communication is realised by electronic interconnects while longer (inter-chip) communication is realized by free space interconnects.

Extensive modelling results for the OTIS have been reported in [10]. The achievable terabit throughput at a reasonable cost makes the OTIS a strong competitor to the electronic alternatives [11, 12]. These encouraging findings prompt the need for further testing of the suitability of the OTIS for real-world parallel applications. A number of studies have been conducted in this direction [3, 11, 13, 14, 16, 18, 19].

In this paper we utilize the OTIS infrastructure to arrive to OTIS-Arrangement. The proposed network will be constructed from the factor network Arrangement. The proposed network and the derived properties can be used in general for all OTIS-Network, which will save the researchers effort to work on each on the OTIS-Networks individually.

In our proposed network we will utilize the OTIS technology in constructing the OTIS_Arrangement as an attractive alternative choice of high speed parallel computing. It is well known that the choice of the network topology for a high-speed parallel computer is an important design decision that involves inherent trade-offs in terms of efficient algorithm support and network implementation cost. However, such networks are difficult to implement using today's electronic technologies that are two dimensional in nature [6, 8]. In principle, free-space optical technologies offer several fronts to improve this trade-off. The improved transmission rate, power consumption, and signal interference are few examples on these fronts [10, 11, 16, 21].

The efficiency of the proposed network comes from taking advantage of the good properties of optical networks including the high speed nature, in addition to the benefit of combining it with the topological properties of arrangement graph including the scalability of selecting the appropriate network size, e.g., OTIS_Star [3].

The arrangement graph as a generalization of the star graph was proposed by day and Tripathi [9], in an attempt to address the scalability problem in the star graph. The arrangement graph slightly improves the scalability problem of the star graph and preserves the desirable properties of this graph. However, since its introduction there has little work done on the development of new algorithms for the arrangement graph and in fact it still inherit the same difficulties in developing efficient routing algorithms that the traditional networks suffers from since it is still a two dimensional network.

As a candidate solution the OTIS_Arrangement network is proposed which is constructed from the graph product of the factor network arrangement. We show the possibility of the proposed OTIS_Arrangement as an underlying topology for parallel computers. The OTIS_Arrangement network improves the scalability problem from which the OTIS_Star and arrangement graph suffer, furthermore it preserves all the attractive features of these two networks.

On the other hand, we show that the OTIS_Arrangement network allows the development of efficient algorithmic structural outlooks that support a wide class of parallel applications, e.g. matrix computation problems, based on the grid and pipeline structures.

2. Graph Definition and Properties

Let n and k be two integers satisfying $1 <= k <= n-1$ and let us denote $<n> = \{1, 2, …., n\}$ and $<k> = \{1, 2, …, k\}$. Let P_k^n taken k at a time, the set of arrangements of k elements out of the n elements of $<n>$. The k elements of an arrangements p are denoted $p_1, p_2, …, p_k$.

Definition 1: the (n, k)-arrangement graph $A_{n,k} = (V,E)$ is an undirected graph given by:

$$V = \{p_1 p_2 \dots p_k \mid p_i \text{ in } <n> \text{ and } p_i \neq p_j \text{ for } i \neq j\} = P_k^n,$$

and

$$E = \{(p, q) \mid p \text{ and } q \text{ in } V \text{ and for some } i \text{ in } <k>, p_i \neq q_i \text{ and } p_j = q_j \text{ for } j \neq i\}.$$

That is the nodes of $A_{n,k}$ are the arrangements of k elements out of n elements of $<n>$, and the edges of $A_{n,k}$ connect arrangements which differ exactly in one of their k positions. For example in $A_{5,2}$ the node p = 23 is connected to the nodes 21, 24, 25, 13, 43, and 53. An edge of $A_{n,k}$ connecting two arrangements p and q which differ only in one position i, it is called i-edge . In this case, p and q is called the (i, q)-neigbour of p. $A_{n,k}$ is therefore a regular graph with degree $k(n-k)$ and $n!/(n-k)!$ nodes. Figure 1 shows $A_{4,2}$ arrangement.

An OTIS based computer contains N^2 processors partitioned into N groups with N processors each. A processor is indexed by a pair $\langle x, y\rangle$, $0 \leq x,y < N$ where x is the group index and y is the processor index.

Processors within a group are connected by certain interconnecting topology, while transposing group and processor indexes achieve inter-group links. Figure 2 shows a 16 processor OTIS connection where the bold dashed arrows represent an optical links between two processors of two different groups. For instance the intra-group connects may be mesh-based, hence the term OTIS-mesh is used to denote this network.

OTIS-Networks are basically constructed by "multiplying" a known topology by itself. The set of vertices is equal to the Cartesian product on the set of vertices in the factor network. The set of edges consists of edges from the factor network and new edges called the transpose edges. The formal definition of OTIS-Networks is given below.

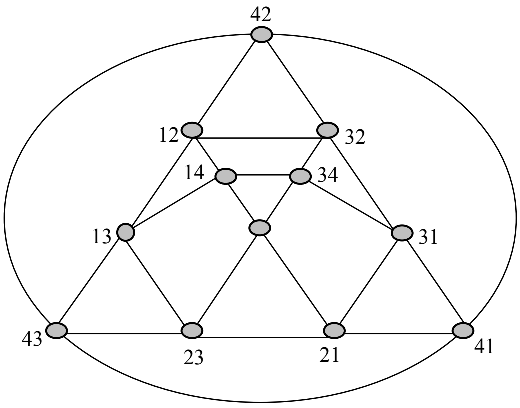

Figure 1. The arrangement graph $A_{4,2}$.

Definition 2: let $G_0 = (V_0, E_0)$ be an undirected graph representing a factor network. The OTIS-$G_0 = (V, E)$ network is represented by an undirected graph obtained from G_0 as follows $V = \{\langle x, y\rangle \mid x, y \in V_0\}$ and $E = \{(\langle x, y\rangle, \langle x, z\rangle) \mid \text{if } (y, z) \in E_0\} \cup \{(\langle x, y\rangle, \langle y, x\rangle) \mid x, y \in V_0 \text{ and } x \mathrel{!=} y\}$.

The set of edges E in the above definition consists of two subsets, one is from G_0, called G_0-type edges, and the other subset contains the *transpose* edges. The OTIS approach suggests implementing G_0-type edges by electronic links since they involve intra-chip short links and implementing transpose edges by free space optics. Throughout this paper the terms "electronic move" and the "OTIS move" (or "optical move") will be used to refer to data transmission based on electronic and optical technologies, respectively.

Definition 2 covers a wide class of OTIS-Networks. In fact, for any known factor network G_0, a new OTIS-network can be obtained by the above definition. The OTIS-Mesh [18], OTIS-Hypercube [13] are only few instances on such networks that have been investigated in the literature.

The Cross Product Network (CPN) as theory framework was proposed by Day and Al-Ayyoub [7] as a tool for generating and defining new interconnection networks and further studying and analysing some of the known interconnection networks. The CPN is basically constructed by "multiplying" two known topologies of the same or different kinds. The cross product network of two interconnection networks given by two undirected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, where $V_1$and V_2 are the set of vertices of G_1 and G_2 and $E_1$and E_2 are the set of edges of G_1 and G_2 respectively. The formal definition of the cross product of the two graphs is defined as follows:

Definition 3: the cross product $G = G_1 \otimes G_2$ of two undirected connected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the undirected Graph $G = (V, E)$, where V and E are given by:

$V = \{\langle x_1, y\rangle \mid x_1 \in V_1 \text{ and } y \in V_2\}$ and

$E = \{(\langle x_1, y\rangle, \langle y_1, y\rangle) \mid (x_1, y_1) \in E_1\} \cup \{(\langle x, x_2\rangle, \langle x, y_2\rangle) \mid (x_2, y_2) \in E_2\}$.

So for any $u = \langle x_1, x_2\rangle$ and $v = \langle y_1, y_2\rangle$ in V, (u, v) is an edge in E if, and only, if either (x_1, y_1) is an edge in E_1 and $x_2 = y_2$, or (x_2, y_2) is an edge in E_2 and $x_1 = y_1$. The edge (u, v) is called a G_1-edge if (x_1, y_1) is an edge in E_1, and it is called G_2-edge if (x_2, y_2) is an edge in E_2 [9]. The size, degree, diameter and number of links of the cross product of two networks are defined next.

Definition 4: if G_1 and G_2 are two undirected connected graphs of respective size s_1 and s_2 and have respective diameters δ_1 and δ_2, then [8, 9]:

1. $G_1 \otimes G_2$ connected.
2. The size s of $G_1 \otimes G_2$ is given by: $s = s_1.s_2$.
3. The diameter δ of $G_1 \otimes G_2$ is $\delta = \delta_1 + \delta_2$
4. The degree of a node $u = \langle x_1, x_2\rangle$ in $G_1 \otimes G_2$ is equal to the sum of the degrees of vertices x_1 and x_2 in G_1 and G_2, respectively.
5. Number of links for the product network, is (size·degree)/2.

3. General Topological Properties

This section discusses some of the basic topological properties of the OTIS-Arrangement network including size, degree, diameter, shortest distance between 2 nodes, and number of links. The topological properties of the OTIS-Arrangement network along with those of the arrangement network are discussed and derived below. These topological properties have been derived using the theoretical framework for analyzing topological properties of the OTIS-Networks in general which was proposed by Al-Ayyoub and Day [7], beside other related research work [8]. Figure 3 shows OTIS-$A_{3,2}$ network which consist of 6 copies of $A_{3,2}$.

We will refer to g as the group address and p as the processor address. An intergroup edge of the form

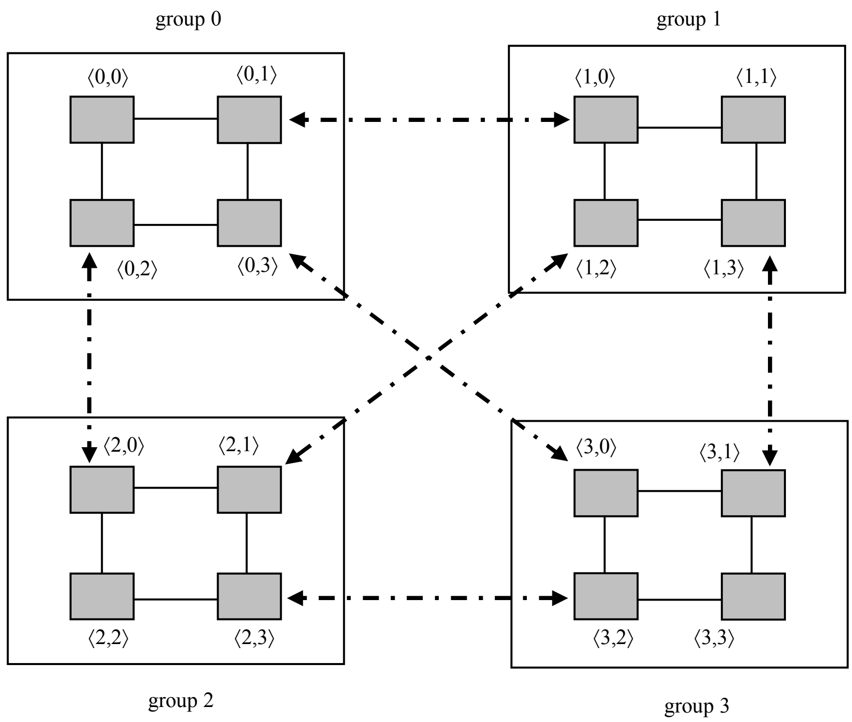

Figure 2. An OTIS connection with 4 groups of 4 processors each.

(⟨g, p⟩, ⟨p, g⟩) represents an optical link and will be referred to as OTIS or optical move. Note that also we will be using the following notations through our paper:

- $|A_{n,k}|$ = size of the graph $A_{n,k}$.
- |OTIS_ $A_{n,k}$| = size of the graph OTIS_$A_{n,k}$.
- Deg. $A_{n,k}(p)$ = Degree of the graph $A_{n,k}$ at node p.
- Deg. OTIS_$A_{n,k}(g, p)$ = Degree of the graph OTIS_ $A_{n,k}$ at node address <g, p>.
- Dist_ $A_{n,k}(p_1, p_2)$ = the length of a shortest path between the two nodes p_1 and p_2 in Arrangement graph.
- Dist. OTIS_$A_{n,k}(p_1, p_2)$ = the length of a shortest path between the two nodes <g_1, p_1> and <g_2, p_2> in OTIS_Arrangement.

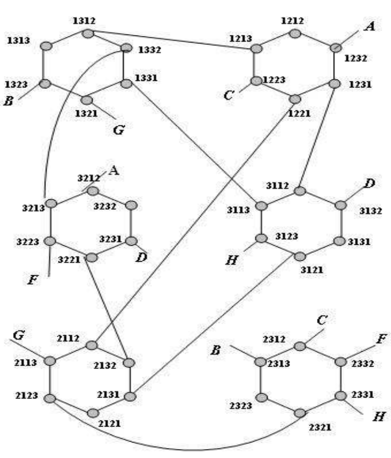

Figure 3. OTIS-$A_{3,2}$ network.

In the OTIS_Arrangement the notation ⟨g, p⟩ is used to refer to the group and processor addresses respectively. Two nodes ⟨g_1, p_1⟩ and ⟨g_2, p_2⟩ are connected if and only if $g_1 = g_2$ and $(p_1, p_2) \in E_0$ (such that E_0 is the set of edges in Arrangement network) or $g_1 = p_2$ and $p_1 = g_2$, in this case the two nodes are connected by transpose edge.

The distance in the OTIS-Arrangement is defined as the shortest path between any two processors, ⟨g_1, p_1⟩ and ⟨g_2, p_2⟩, and involves one of the following forms:

- When $g_1 = g_2$ then the path involves only electronic moves from source node to destination node.
- When $g_1 \neq g_2$ and if the number of optical moves is an even number of moves and more than two, then the paths can be compressed into a shorter path of the form: $\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, p_2\rangle \xrightarrow{O} \langle p_2, g_1\rangle \xrightarrow{E} \langle p_2, g_2\rangle \xrightarrow{O} \langle g_2, p_2\rangle$ here the symbols O and E stand for optical and electronic moves respectively.
- When $g_1 \neq g_2$, and the path involves an odd number of OTIS moves. In this case the paths can be compressed into a shorter path of the form: $\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, g_2\rangle \xrightarrow{O} \langle g_2, g_1\rangle \xrightarrow{E} \langle g_2, p_2\rangle$.

The following are the basic topological properties for the OTIS_Arrangement. For instance if the factor Arrangement network is of size $n!$, degree is n-1 and diameter is $\lfloor 1.5\ k \rfloor$ [9]. Then the size, the degree, the diameter, number of links, and the shortest distance of OTIS_Arrangement network are as follows:

- Size of |OTIS_$A_{n,k}$| = $|n!/(n-k)!|^2$.
- Degree of OTIS_$A_{n,k}$ = Deg.($A_{n,k}$), if $g = p$.
- Deg. ($A_{n,k}$) + 1, if $g \neq p$.
- Diameter of OTIS_$A_{n,k}$ = $2\lfloor 1.5\ k \rfloor + 1$.
- Number of links: let N_0 be the number of links in the $A_{n,k}$ and let M be the number of nodes in the $A_{n,k}$. The number of links in the OTIS_$A_{n,k}$ $= (M^2 - M)/2 + N_0^2$. For instance, the number of links in the OTIS_$A_{4,2}$ consisting of 144 processors is $= (12^2 - 12)/2 + 23_0^2 = 595$.
- Dist. of OTIS-An, k = $\begin{cases} \min\left(d(p1, g2) + d(g1, p2) + 1, d(p1, p2) + d(g1, g2) + 2\right) & if g1 \# g2 \\ Dist\,(p1, p2) & if g1 = g2 \end{cases}$

Theorem 1: the length of the shortest path between any two processors $\langle g_1, p_1\rangle$ and $\langle g_2, p_2\rangle$ in OTIS-Arrangement is $d\,(p_1, p_2)$ when $g_1 = g_2$ and $\min\{d\,(p_1, p_2) + d\,(g_1, g_2) + 2, d\,(p_1, g_2) + d\,(g_1, p_2) + 1\}$ when $g_1 \neq g_2$, where $d\,(p, g)$ stands for the shortest distance between the two processors p and g using any of the possible shortest paths as seen in the above forms 1, 2 and 3 [1].

It is obvious from the above theorem that when $g_1 = g_2$, then the length of the path between the two processors $\langle g_1, p_1\rangle$ and $\langle g_2, p_2\rangle$ is $d\,(p_1, p_2)$. From the shortest path construction methods in (2) and (3) above, it can be easily verified that the length of the path equal min $\{d\,(p_1, p_2) + d\,(g_1, g_2) + 2, d\,(p_1, g_2) + d\,(g_1, p_2) + 1$ when $g_1 \neq g_2\}$.

To send a message M from the source node $\langle g_1, p_1\rangle$ to the destination node $\langle g_2, p_2\rangle$ it must follow a route along one of the three possible paths *1*, *2,* and *3*. The length of the shortest path between the nodes $\langle g_1, p_1\rangle$ and $\langle g_2, p_2\rangle$ is one of the forms:

$$\text{Length} = \begin{cases} d\,(p_1, p_2) & \text{if } g_1 = g_2 \\ min\,(\,d\,(p_1, g_2) + d\,(g_1, p_2) + 1, d\,(p_1, p_2) + d\,(g_1, g_2) + 2) & o.w. \end{cases} \quad (1)$$

Where $d\,(p_1, p_2)$ is the length of the shortest path between any two processors $\langle g_1, p_1\rangle$ and $\langle g_1, p_2\rangle$. If δ_0 is the diameter of the factor network $A_{n,k}$ then from (I) it follows that the diameter of the OTIS_$A_{n,k}$ is $2\delta_0 + 1$.

The diameter of OTIS_$A_{n,k}$ is the *Max* (δ_0, $2\delta_0 + 1$) which is equal to $2\delta_0 + 1$. The proof of the above theorem is a direct result from (I).

4. Routing in OTIS-Arrangement Network

In this section we present a new routing algorithm for the OTIS_Arrangement networks called Route_OTIS_$A_{n,k}$. In order to clarify the algorithm we present first the following definitions:

Definition 5: let $c^i_{A_{n,k}}$ be the i^{th} element of the current node in the OTIS_$A_{n,k}$ graph representing a factor network, where $1 \leq i \leq k$ and $1 \leq c^i_{A_{n,k}} \leq n$.

Definition 6: let $d^i_{A_{n,k}}$ be the i^{th} element of the destination node in the OTIS_$A_{n,k}$ graph representing a factor network, where $1 \leq i \leq k$ and $1 \leq d^i_{A_{n,k}} \leq n$.

Definition 7: let $N^i_{A_{n,k}}$ be the direct neighbor of the current node $c^i_{A_{n,k}}$ via a replacement move of changing only the i^{th} element of $c^i_{A_{n,k}}$ chosen out of the n elements except the elements that are equal to the values of the k elements beside $c^i_{A_{n,k}}$.

We now present the routing algorithm Route_OTIS_ $A_{n,k}$ to find the nodes following a current node $<g_c, p_c>$ towards a destination node $<g_d, p_d>$ via a minimal distance routing.

*Algorithm Route_OTIS_* $A_{n,k}$ ($<g_c, p_c>$, $<g_d, p_d>$)
// routing from the current node $<g_c, p_c>$ *toward the destination node* $<g_d, p_d>$
{
if $g_c = g_d$ *and* $p_c = p_d$ *then stop //destination reached*
if $g_c = g_d$ *then Route_OTIS_* $A_{n,k}$ ($<g_d$, *Route_* $A_{n,k}$ $<p_c, p_d>$, $<g_d, p_d>$)
*if (dist_* $A_{n,k}$ (p_c, p_d) + *dist_* $A_{n,k}$ $(g_c, g_d) + 2) <$ (*dist_* $A_{n,k}$ (p_c, g_d) + *dist_* $A_{n,k}$ (g_c, p_d)) *then*
if $p_c = p_d$ *then Route_OTIS_* $A_{n,k}$ ($<p_c, g_c>$, $<g_d, p_d>$)
*else Route_OTIS_* $A_{n,k}$ ($<g_c$, *Route_* $A_{n,k}$ $<p_c, p_d>$, $<g_d, p_d>$)
if $p_c = g_d$ *then Route_OTIS_* $A_{n,k}$ ($<p_c, g_c>$, $<g_d, p_d>$)
*Route_OTIS_* $A_{n,k}$ ($<g_c$, *Route_* $A_{n,k}$$<p_c, g_d>$, $<g_d, p_d>$})
*Function Route_* $A_{n,k}$ $< c_{\mathrm{An,k}}, d_{\mathrm{An,k}} >$
// moving from the current node $c_{\mathrm{An,k}}$ *toward the destination node* $d_{\mathrm{An,k}}$ *via a neighboring*
// node $N_{\mathrm{An,k}}$
{ *for i=1 to k*
{
select a neighbor $N^i_{A_{n,k}}$ *where dist_* $A_{n,k}$ $(N^i_{A_{n,k}}, d^i_{A_{n,k}}) <$ *dist_* $A_{n,k}$ $(c^i_{A_{n,k}}, d^i_{A_{n,k}})$ *// choose an optimal neighbor via the* i^{th} *dimension out on the n elements among the shortest path toward the destination*
}
}

This routing algorithm checks first the related locations of the source and the destination nodes. If both nodes are in the same group; the same factor $A_{n,k}$ network; then the algorithm corresponds to a series of electronic moves transforming p_c to p_d along a minimal path in the $A_{n,k}$ as generated by the minimal distance routing function Route_ $A_{n,k}$. If the source node is located in different group than the destination node's group then there are two cases.

In the first case, if (dist_ $A_{n,k}$ (p_c, p_d) + dist_ $A_{n,k}$ (g_c, g_d) + 2) < (dist_ $A_{n,k}$ (p_c, g_d) + dist_ $A_{n,k}$ (g_c, p_d)) then the routing path must have two optical moves; as explained in the previous section; the algorithm first repeatedly transforms p_c to p_d along a minimal distance path in the $A_{n,k}$ generated by Route_ $A_{n,k}$. In a second stage, the algorithm makes an optical move to node $<p_d, g_c>$. Then, the algorithm repeatedly transforms g_c to g_d along a minimal distance path in the $A_{n,k}$ generated by Route_ $A_{n,k}$. Finally, the algorithm makes the second optical move to reach the destination node.

In the second case, if (dist_ $A_{n,k}$ (p_c, p_d) + dist_ $A_{n,k}$ (g_c, g_d) + 2) ≥ (dist_ $A_{n,k}$ (p_c, g_d) + dist_ $A_{n,k}$ (g_c, p_d)) then the routing path has one optical move, the algorithm first repeatedly transforms p_c to g_d along a minimal distance path in the $A_{n,k}$ generated by Route_ $A_{n,k}$. In a second stage, the algorithm makes an optical move to node $<g_d, g_c>$. Finally, the algorithm repeatedly transforms g_c to p_d along a minimal distance path in the $A_{n,k}$ generated by Route_ $A_{n,k}$.

The function Route_ $A_{n,k}$ performs a simple rule to find a routing path within the $A_{n,k}$. It selects a direct neighbor via the i^{th} dimension out on the n elements where this direct neighbor is among one of the shortest paths toward the destination. This selection is performed by a replacement move of changing only the i^{th} element of the current node chosen out of the n elements except those elements that are equal to values of other k elements beside the i^{th} element in the current node. This rule insures a minimal distance path. Furthermore, it will allows us to calculate the length of this path by examining the selection involved.

Example 1: consider the Route_OTIS_ $A_{n,k}$ routing algorithm to route a message from the source node <23, 12> to the destination node <23, 21> in an OTIS_ $A_{4,2}$ network.

Since both nodes are in the same group; the same factor $A_{4,2}$ network; then the algorithm corresponds to a series of electronic moves transforming 12 to 21 along a minimal path in the $A_{4,2}$ as generated by the minimal distance routing function Route_ $A_{4,2}$. These moves are as follows:

$$\langle 23, 12\rangle \xrightarrow{E} \langle 23, 14\rangle \xrightarrow{E} \langle 23, 24\rangle \xrightarrow{E} \langle 23, 21\rangle.$$

Example 2: Consider the Route_OTIS_ $A_{n,k}$ routing algorithm to route a message from the source node <13, 23> to the destination node <12, 23> in an OTIS_ $A_{4,2}$ network.

Since both nodes are in the different groups and (dist_ $A_{n,k}$ (p_c, p_d) + dist_ $A_{n,k}$ (g_c, g_d) + 2) < (dist_ $A_{n,k}$ (p_c, g_d) + dist_ $A_{n,k}$ (g_c, p_d) then the routing path must has two optical moves. The routing path will be $\langle 13, 23\rangle \xrightarrow{O} \langle 23, 13\rangle \xrightarrow{E} \langle 23, 12\rangle \xrightarrow{O} \langle 12, 23\rangle$.

5. Conclusions

In this paper, we have contributed to the study of OTIS and Arrangement networks in general and to OTIS_$A_{n,k}$ in specific through proposing the new OTIS-Arrangement network as an attractive network to its factor network. Several topological properties including size, degree, diameter, number of links and shortest distance between any two nodes have been derived. An efficient algorithms: routing has been proposed for the OTIS_$A_{n,k}$. As a future research work we can utilize the proposed algorithms in solving real life problems on OTIS_$A_{n,k}$ including Matrix problems and Fast Fourier transforms. The proposed OTIS_$A_{n,k}$ could be an attractive alternative for its factor network in terms of routing by utilizing both electronic and optical technologies.

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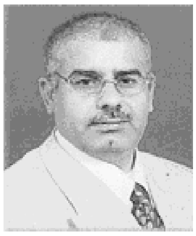

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