Designing an Intelligent Recommender System Using Partial Credit Model and Bayesian Rough Set

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Abstract: *Recommender systems have become fundamental in web-based applications and information access. They effectively prune large information spaces and provide appropriate decision making and suggestions so that users are directed toward those items that best meet their needs, preferences and interests. In web-based learning context, these systems usually neglect the learner's ability, the difficulty level of the recommended item (e.g., learning resource, exam), and the learner self-assessment. Therefore, this paper suggests an intelligent recommendation system to provide adaptive learning. The suggested system consists of two main intelligent agents. First, a personalized learning resource based on partial credit model (PLR-PCM) agent which considers both the learner's ability and the learning resource difficulty to provide individual learning paths for learners. Second, BRS-Recommendation agent provides decision rules as instrument or guide for the learner's self-assessment using Bayesian Rough Set (BRS), based on inductive learning algorithm. Experimental results show that the proposed system can exactly provide a learning resource closer to the learner's ability with appropriate feedback to the learner, resulting in the improvements of the learning efficiency and performance.*

Keywords: *Recommender systems, partial credit model, inductive learning algorithm, bayesian rough set.*

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1. Introduction

Learning management system follows to deliver a specific learning resource to the learners, as intended by the content developer. This learning resource may be free choice mode, which indicates that the learner is free to choose any activity in any order without restriction, or it may be guided by flow through the structure of the content organization. Despite these capabilities, it still suffers from a lack of personalization, which may lead to inappropriate learning to the learner. Therefore, we propose a Personalized Learning Resource Based on Partial Credit Model (PLR-PCM) to overcome above drawbacks. Nevertheless, the rising question is "where should the learner put his or her efforts to obtain the necessary background knowledge for the next learning resource?" If the answer to this question can be found automatically based on the outcomes of previous learners, then the learners, on one hand, will be able to focus on learning what they need, the content developer, on the other hand, will be able to reorganize the sequence of the learning resources in an appropriate order. To meet the requirements, in this paper, we also discuss how to use Bayesian Rough Set (BRS) based inductive learning to assist the learners, the teachers and the content developers in the web-base learning.

In the last few years, some researchers have focused on the recommendation systems, mainly addressing to the technology of generating recommendations, such as the application of data mining techniques [14]. Others have given considerable attention to personalize web-based learning using Item Response Theory (IRT). For example, Chen *et al.* [6] extended the dichotomous IRT based on fuzzy set, so that the learning system can recommend courseware with appropriate difficult levels to the learners, and the learners can give a fuzzy response about the understanding percentage of the learned courseware. Chen *et al.* [7] proposed a personalized e-learning system based on dichotomous IRT. Thereafter, he and his colleagues have presented a prototype of personalized web-based instruction system (PWIS) based on modified IRT to perform personalized curriculum sequencing by simultaneously considering the courseware difficulty level, the learner's ability and the concept continuity of learning pathways during learning process [5].

This study proposes a PLR-PCM which takes into account the learner's ability and the difficulty of learning resource. In order to allow more information about the level of ability to be extracted from a fixed set of learning resources, the estimation of this ability does not depend only on the learner's explicit response, but also depend on learner's implicit responses which are related to the learner's ability. Therefore, the proposed system can prevent learners from being lost in the learning resources or surfing from distraction or restriction, by providing personalized learning guidance, filtering out inappropriate learning resources to reduce cognitive loading, and providing a fine learning diagnosis based on an individual's user profile. Moreover, BRS based inductive learning improves the state-of-the-art of web-based learning by providing intelligent feedback and making learning more effective.

2. An Intelligent Recommender System Architecture

In this section, an adaptive learning approach for the proposed system based on the PCM and BRS, which includes four intelligent agents and two databases, is presented herein. The four intelligent agents are the learner's interface agent, the learner's feedback agent, the PLR-PCM agent and the BRS-recommendation agent, respectively. These two databases include the learner profile database and courseware database. The learner interface agent aims at providing a flexible learning interface for the learners to interact with the learner's feedback agent, PLR-PCM agent and BRS-recommendation. The learner's feedback service aims to assemble both explicit and implicit responses, update the learner's ability and evaluate the difficulty parameters of learning resources. The PLR-PCM agent aims to recommend suitable learning recourses with appropriate difficulties to the learners, using PCM and deciding on the next activity to be recommended to the learners, according to learner feedback response. The BRS-recommendation agent aims to provide general recommendation rules to the learners and the content developers, using improved BRS. The system architecture is shown in Figure 1.

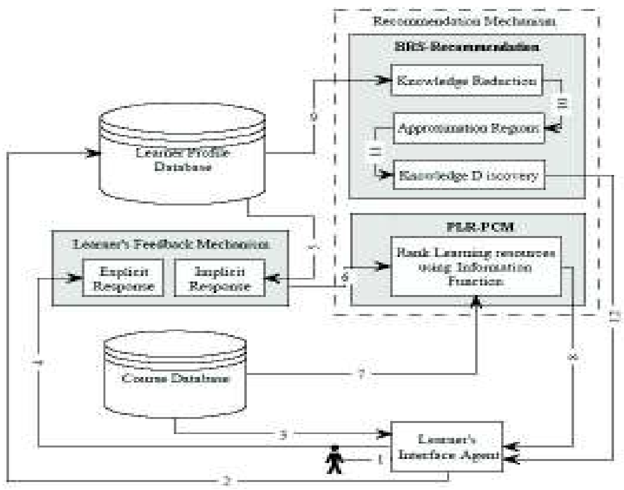

Figure 1. The proposed system architecture.

3. Learner's Feedback Agent

The learner's feedback agent collects explicit responses from the learner's interface agent and simultaneously collects implicit responses from the learner's profile database; these responses are explained as follow:

- *Explicit feedback response*: A direct learner's response (multiple choices) which contains only one question associated with understanding the learning resource. This question has three understanding levels, 'low understanding', 'moderate understanding' and 'high understanding'.
- *Implicit feedback responses*: An indirect learner's response that is associated with the learner's characteristic responses. Below is a description for each implicit response:
  1. *Learning time response*: Is the sum of all of the learner's session times.
  2. *Learning score response*: Is the learner's score for the specific learning resource.
  3. *Learning attempt*: Is the number of attempts on the activity.

3.1. Tuning Implicit Reponses

Learner's ability estimation does not depend only on the understanding level, but also depends on the implicit responses, because they have a direct impact on learner's ability. For example, if learner completes learning resource in short time with a small number of attempts and obtains a high score, then his or her ability is increased. In contrast, if the learner completes the learning resource in a long time with a high number of attempts and obtains a low score then his or her ability is decreased. Once these responses are collected by the learner's feedback mechanism, they are converted to the response categories. Therefore, this study firstly proposes a general form of an Implicit Response Rate (*IRR*), which converts actual response values into values limited to between 0 and 1, as demonstrated by the following relationship:

$$IIR = \frac{\textit{Implicit response}}{\textit{Maximum information allowed}} \tag{1}$$

Where *implicit response* is either learning time, attempt or score divided by the maximum time, attempt or score allowed, respectively. Thereafter, the implicit response category j is calculated using the following equation:

$$m = \begin{cases} 2\,(A3)\,w_5 \le IRR \le w_6 \\ 1\,(A2)\,w_3 \le IRR \le w_4 \\ 0\,(A1)\,w_1 \le IRR \le w_2 \end{cases} \tag{2}$$

Where $A1$, $A2$ and $A3$ denote verbal response categories, which are obtained from Table 1, and $w_1, w_2, \ldots, w_6$ denote the limiting values. Calculating m associated with the learning score using the above equation, where as $w_1=0$, $w_2=\alpha$, $w_3=\alpha$, $w_4=1\text{-}\alpha$, $w_5=1\text{-}\alpha$ and $w_6=1$. Furthermore, calculating m associated with learning time or attempt also using the above equations, whereas w_1=1-α, w_2=1, w_3=α, w_4=1-α, w_5=0 and w_6=α. Usually, α is in the interval (0, 0.5).

Table 1. Verbal response categories.

	A1	A2	A3
Time	Long	Medium	Short
Attempt	Small	Moderate	High
Score	Low	Average	High

The system presented here can use equations 1 and 2 to automatically tune the implicit responses, after that, the tuned implicit responses (numeral response categories) are sent to the learner's ability algorithm in order to estimate learner's ability.

3.2. Learner's Ability Estimation

IRT models are applied to the items that are scored polytomously (more than two responses). If the options of the rating scale are successfully ordered as in the Likert scale [10], then the Rasch family of the polytomous IRT models, including the Partial Credit Model (PCM) [8], Rating Scale Model (RSM) [4] and the Graded Response Model (GRM) [12, 13].

The polytomous IRT model is sometimes referred to as the RSM when all items have the same number of the threshold categories, and the difference between any given threshold category and the mean of the threshold locations is equal or uniform across the items, where RSM is a special case of the PCM. The model is also sometimes referred to as the PCM, particularly when applied in Computerized Adaptive Testing (CAT) [11].

The learner's ability algorithm estimates the level of a latent trait of the learner demonstrated in an observed polytomous response pattern. In considering polytomous data, it is convenient to treat the coding and scoring operations separately. For the jth polytomous item, we will code n possible response categories with the arbitrary labels $x_{j0}, x_{j1},\ldots, x_{j(n-1)}$, and indicate the learner's response with the (random) response variable $X_j \in x_{j0}, x_{j1},\ldots, x_{j(n-1)}$.

There exists ability variable θ such that a response in a higher category reflects a higher ability level than a response in a lower category. According to The PCM, the probability of scoring in a response category m on item j is given by a response function $P_{jm}(\theta)$ as shown in the following equation:

$$P_{jm}(\theta) = \frac{\exp \sum_{h=0}^{m} (\theta - b_{jh})}{\sum_{m=0}^{n-1} \exp \sum_{h=0}^{m} (\theta - b_{jh})} \tag{3}$$

The location parameter b_{jm} can be broken down further to indicate the item's difficulty and the category threshold, i.e., $b_{jm}=b_j+d_{jm}$. The difference in levels of proficiency between the adjacent x_{jm} and x_{jm+1} categories is called step difficulty or threshold d_{ij}. The step difficulty is also equal the intersection point between two adjacent curves. The joint maximum likelihood estimation (JMLE) is used for estimate the learner's ability, which maximizes the likelihood function for a particular response [16]. In equation 4, the likelihood function $L(X_j=x_j|\theta)$ can be further described as follows:

$$L(X_j = x_j \mid \theta) = \prod_{j=1}^{J} \prod_{m=0}^{n-1} P_{jm}(\theta)^{y_{jm}} \tag{4}$$

Where y_{jm} = 1 when $x_j = X_{jm}$, and 0 otherwise (i.e., y_{jm} are the observed values of y_{jm}). Similar to the item parameter estimation, the learner's ability estimation is an iterative process. It begins with some a priori value for the ability of the examinee. It begins with θ= 0.

In this study, we assume that each learning resource has three responses: learning time, attempt and score response. The randomly chosen learner responds to a set of *Jth* learning resources with an array of response category patterns $P = (p_1, p_2,\ldots, p_j,\ldots,p_J)$, where p_j is a response category pattern, each of which has three response categories, which are labeled either 0, 1 or 2. Thus, the total number of item responses in the activity tree is equal to *J*3*.

4. PLR-PCM Agent

Information functions indicate how much information or precision of measurement an item or test provides, conditional on the learner's ability level, and are useful for describing, comparing and selecting items. A polytomous information function, however, can be estimated for each category as well as for the entire item [8]. Equation 5 is applied to compute the matched degree for recommending an appropriate learning resource to the learners:

$$I(\theta) = \left(\sum_{m=0}^{n-1} m^2 P_{jm} \right) - \left(\sum_{m=0}^{n-1} m P_{jm} \right) \tag{5}$$

Where θ denotes new learner ability estimated after *Jth* preceding learning resources, P_{jm} represents the probability of m response category.

That is, the PLR-PCM agent provides comprehensive learning experiences to the learners, such as knowledge sequencing, concept sequencing and learning sequencing. Knowledge sequencing recommends the next learning resource; concept sequence recommends the next learning activity such as course, lesson and module, and the learning sequence gives a sequence of the learning activities.

5. BRS-Recommender Agent

In web based-application, the recommendation rules are used to help web users to find out pages which are interested to them based on information entropy of data in positive domain to help finding out the rough set rules [20]. In [17], an intelligent recommendation approach named Recommendation by Rough-Set and Collaborative Filtering (RSCF), which integrates collaborative information and content features, is proposed to predict user preferences on the basis of rough-set theory. In [1], a new model based on BRS is proposed to analyze a real learner information database to find the recommendation rules behind the information table; that is, by using modified BRS based inductive learning algorithm, the learning resource that requires more effort will be identified to obtain the necessary background for the next learning resource. Decision rules are obtained using Rough Set based learning to predict academic performance, and it is used to recommend a path for course delivery [9].

Web-based learning does face a number of deficiencies as a consequence of the delivery distance form of teaching. Lack of direct contact and immediate feedback between the learners and the teachers is one of the main problems in distance education [3]. For this reason, we propose a new algorithm to overcome these problems involved. This algorithm is derived from the modified BRS based inductive learning algorithm; however, inductive learning and classification of objects from data sets are important research areas in AI [18]. It has been used to model the knowledge of human experts by using a method to infer decision rules from a sample of expert decisions.

Three steps are defined to extract suitable decision rules from decisions table, as described in the following subsections.

5.1. Attributes Reduction

The attributes reduction is a process of omitting unnecessary condition attributes from the decision table. In other word, not all condition attributes are necessary to categorize the objects in the information system. Some attributes may be redundant or dispensable with respect to the decision attributes D.

Let R_C and R_D be families of equivalence classes and the positive region of D denoted by POS_C (D). $A \in C$ is D-Dispensable in condition attribute C if it satisfies the condition $POS_{C-\{A\}}(D)=POS_C(D)$.

5.2. Approximation Regions

The original rough set addresses the case where there exist fully correct. In addition, there is a problem associated with rough set theory, is the lower approximation (positive regions), will always be empty if uncertainty (boundary regions) widely exists. Therefore, we modify BRS model based on Bayesian Confirmation Measures to enhance the precision of original rough set and to deal with both two decision classes and multi decision classes [2].

After recalling the basic methods for extracting probabilities from data, the improved BRS Positive, Negative, and Boundary regions for multi decision classes are, respectively defined, where $r(X,E)=\log\frac{P(X|E)}{P(E)}$ is the relevance measure, when evidence E confirms hypothesis X, X_t is the target concept (positive hypothesis), X_c the complement concept (negative hypothesis), P (X| E) is the posterior probability, and β=log (1-α), α Є [0, 1).

$$\begin{aligned} POS(X_t)^{\alpha} &= \cup\left\{\left(E_i:\forall_{c:c\neq t}\; r(X_c,E_i)\leq\beta\right)\right.\\ &\qquad \left.\vee\left(P(X_c\,|\,E_i)=0\right)\right\}\\ NEG(X_t)^{\alpha} &= \cup\left\{\left(E_i:r(X_t,E_i)\leq\beta\right)\right.\\ &\qquad \left.\vee\left(P(X_t\,|\,E_i)=0\right)\right\}\\ BND(X_t)^{\alpha} &= \cup\left\{\left(E_i:\exists_{c:c\neq t}\, r(X_c,E_i)>\beta\right)\right.\\ &\qquad \left.\wedge\left(E_i:r(X_t,E_i)>\beta\right)\right\} \end{aligned} \tag{6}$$

5.3. Extraction of Decision Rules Using Certainty Measures

During learning experiences, decision rules can be used to guide the learners. For repeating a specific learning resource, these rules notify them of the activities that need to be emphasized for the second round of the study. Meanwhile, the new learners are informed about these activities where extra efforts need to be put in to pass the post test (final exam).

However, we use discriminant index to provide a measure of the degree of certainty in classifying the set of objects, whereas the highest index value determines the best attribute [19]. Equation 7 is applied to compute the discriminant index:

$$\eta = 1-\frac{card\,(BND^{\alpha}(X_t))}{card\,(U)} \tag{7}$$

Where, η is the discriminant index, *card* is the cardinality of the set and U is the universal set.

6. Experiments

In order to provide the proof of feasibility study and verify the learning performance for the proposed system, we used a SCORM 2004 Photoshop example version 1.1 [15], it contains a collection of the learning resources to do with Adobe Photoshop, the manifest file (named as 'imsmanifest.xml') and other relevant files. Moreover, it illustrates several sequence ways based on the same course content using various instructional strategies. The details of the system functions and experimental results are described as follows:

a. *The System Implementation*: The proposed system has been applied in the labs of computer science and all results have been collected from applying SCORM 2004 Photoshop example version 1.1 [15] in which learners used learning management system. The proposed system was successfully implemented using Microsoft Windows XP Service Pack 2 with Apache Web Server version 2.2.6, PHP script language version 5.2.5 and MySQL database version 5.0.45.When the learner logs into the system, the learner's interface is started, as shown in Figure 2. The left frame shows an activity tree hierarchy of the course concepts on a Photoshop subject, the upper part of the right frame shows the content of the learning resource, and the lower part of the right frame shows the learner feedback interface corresponding to the content shown in the upper part.

Figure 2. Learner's interface.

During the learning experience, learner gives polytomous responses according to the type of learning resources (module-lesson or module-exam). When the learner completes the content of learning resource, the feedback agent collects the learner's responses and re-evaluates the learner's ability by considering the proposed system and assigns moderate abilities for beginner learners. Table 2 depicts a list of learning sequence recommendations based on a moderate learner's ability; it includes three attributes: Priority P indicates the priority of learning resources, learning activity sequence includes the sequence of recommended learning resources and sequence information value I (θ) denotes the recommended degrees of the learning sequences.

b. *Experimental Results and Evaluation*: In this system, the range of both the learner's ability and learning resource difficulty are limited to between -3, which is very weak ability or very easy material, to 3, which is very high ability or very difficult material.

The learner's ability is increased if he or she can complete the content of the learning resource with a high degree of understanding or high score, a short session time and a small number of attempts. In contrast, the learner's ability is decreased, if he or she completes the content of the learning resource with a low degree of understanding or low score, a long session time and a large number of attempts. Figure 3 presents four ability curves of the four learners; each learner has different learning sequences with different abilities.

Table 2. Learning sequence recommendation based on moderate learner's ability.

P	Learning Activity Sequence	I(θ)
1	Module 1-Basics → Lesson 3-Palettes	0.4237
2	Module 1-Basics → Lesson 1-Interface	0.4231
3	Introduction	0.4228
4	Module 1-Basics → Lesson 2-Toolbox	0.4223
5	Module 2-Enhancing Images → Lesson 5-Color Balance	0.4136
6	Module 2-Enhancing Images → Lesson 7-Hue/ Saturation	0.4112
7	Module 2-Enhancing Images → Lesson 6-Brightness / Contrast	0.4078
8	Module 1-Basics → Module 1-Exam → Question 3	0.3678
9	Module 1-Basics → Module 1-Exam → Question 2	0.3644
10	Module 1-Basics → Module 1-Exam → Question 1	0.3532
11	Module 1-Basics → Lesson 4-Layers	0.3501
12	Module 2-Enhancing Images → Module 2-Exam → Question 4	0.3110
13	Module 2-Enhancing Images → Module 2-Exam → Question 5	0.3065
14	Module 2-Enhancing Images → Module 2-Exam → Question 6	0.2958
15	Module 3-Blending Images → Lesson 8-Selection Tools	0.1831
16	Module 3-Blending Images → Lesson 9-Transform	0.1671
17	Module 3-Blending Images → Module 3-Exam → Question 7	0.1537
18	Module 3-Blending Images → Module 3-Exam → Question 8	0.1509
19	Module 3-Blending Images → Module 3-Exam → Question 9	0.1470

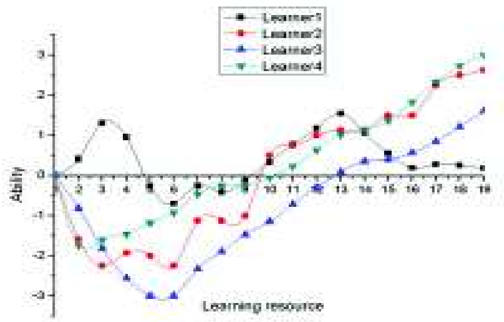

Figure 3. Ability curves of the four learners.

The Learner's ability is dramatically tuned in the initial stage. When learners learn the appropriate learning resources that are recommended by the system during a learning process, their abilities will gradually approach a stable value. Table 3 indicates that the proposed system first recommends the learning resource 'Lesson 3-Palettes' to the learner, because this learning resource, with difficulty parameter 0.0938 is the most difficult one close to ability 0. Next, the learner is assumed to answer the question 'what do you think about your understanding level?' with 'moderate understanding'; thus, he or she completes the content of the current learning resource with a long session time, a large number of attempts and a moderate degree of understanding. After the learner presses the button 'recommend me', the ability is decreased to -

1.7368; then the system recommends the next learning resource 'Question 1' with difficulty parameter -1.5000. Noticeably, the proposed system might recommend the learning resource 'Question 1' instead of 'Introduction' to the learner at the beginning of the learning process; nevertheless he or she might not have enough knowledge to answer this question. This situation is logical because, firstly, learner's ability is closer to the difficulty of that learning resource and, secondly, the Photoshop example was authored as free to choose with out any relation degree among learning resources. However, the implicit responses of the module-exams (Questions 1 to 9) are either 0 or 2, because these questions contain only true-false question. By taking this into account, when any learning resource is satisfied and completed, the system must delete it from the recommendation list. This process will continue until the learner satisfies all learning resources or he/she logs out of the system.

Table 3. Learner's characteristics for whole course.

Id	Learning Resource Title	T	A	U/S	Difficulty	Ability
1	Lesson 3-Palettes	0	0	1	0.0938	-1.7368
2	Question 1	1	1	2	-1.5000	-1.5917
3	Question 2	1	1	2	-1.2500	-1.4642
4	Question 3	2	2	0	-1.1667	-1.1744
5	Lesson 6-Brightness	2	2	0	-0.7500	-0.9217
6	Lesson 7-Hue/Saturation	2	2	2	-0.6667	-0.4667
7	Lesson 5-Color Balance	2	2	1	-0.6000	-0.2473
8	Lesson 2-Toolbox	1	1	1	-0.2400	-0.2463
9	Lesson 1-Interface	2	2	1	-0.1667	-0.0483
10	Introduction	2	2	2	0.1364	0.2348
11	Lesson 4-Layers	2	2	2	1.7000	0.6415
12	Question 4	2	2	2	1.7813	1.0223
13	Question 5	2	2	0	1.8750	1.1455
14	Question 6	2	2	2	2.1000	1.3717
15	Lesson 8-Selection Tools	2	2	2	2.2500	1.8346
16	Question 7	2	2	2	2.5000	2.3396
17	Question 8	2	2	2	2.6250	2.7371
18	Question 9	2	2	2	2.7000	3.0000

Where *T*, *A*, *U* and *S* are a learning time, learning attempt, learning understanding level and learning score, respectively.To evaluate the performance of the proposed system, three experiences were performed with three sequence modes: a 'recommended choice' mode means learner can click on 19 recommended learning resources. A 'sequential choice' mode means learner must experience and complete the 'introduction' first, then all modules and lessons in a linear order directed by the proposed system. Finally in 'free choice' mode, the learner can 'jump' to (select) individual lessons and specific modules in any order regardless of any system recommendation. According to these modes, Figures 4-6 present the relationship between difficulty of the learning resource and learner's ability, respectively.

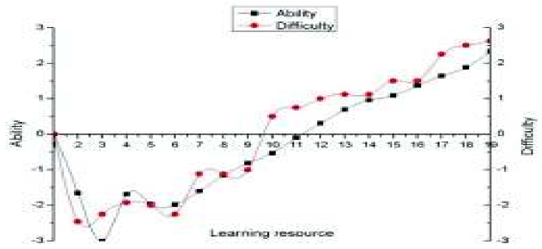

Figure 4. Recommended choice mode.

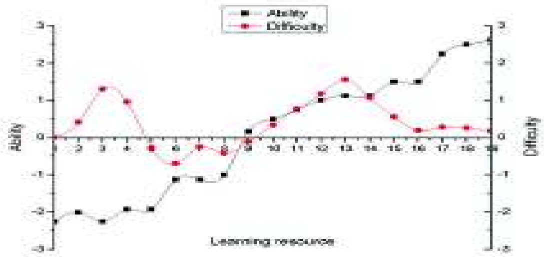

Figure 5. Sequential choice mode.

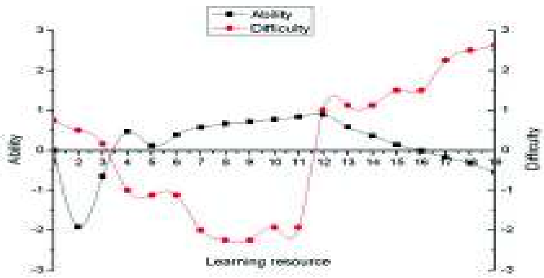

Figure 6. Free choice mode.

A paired t-test was applied to investigate the statistical difference between the difficulty of the learning resource and the learner's ability at significance level 0.05. The results are shown in Tables 4-6. From these Tables, it can be seen that the P-value for experiments 1, 2 and 3 are 0.02319, 0.43913 and 0.87564, respectively. This indicates that a higher correlation was found with experiment 1, whereas a lower one was obtained with examples 2 and 3. The results also reveal that the standard deviation between the analysis groups (i.e., difficulty and ability) in experiment 1 is much closer. That is, the experimental results show that the proposed system can recommend a suitable learning experience to the learner with high degree of correlation between the learner's ability and the recommended learning resource, resulting in both increased the learning efficiency and learning performance.

Table 4. Paired t-test for recommended choice mode.

Data	Mean	Variance (SD^2)	N	Std. Deviation	Std. Error	Minimum
Difficulty	-0.222	2.361	19	1.536	0.352	-3.000
Ability	0.038	3.023	19	1.738	0.398	-2.456
t = -2.4813, p= 0.02319 (lower significant level) = higher correlation coefficient.						

Table 5. Paired t-test for sequential mode.

Data	Mean	Variance (SD^2)	N	Std.Deviation	Std. Error	Minimum
Difficulty	0.07550	2.89231	19	1.70068	0.39016	-2.25000
Ability	0.38362	0.38672	19	0.62187	0.14267	-0.69556
t = -0.7912, p = 0.43913 (lower to medium significant level) = lower to medium correlation.						

Table 6. Paired t-test for freely choice mode.

Data	Mean	Variance (SD^2)	N	Std.Deviation	Std. Error	Minimum
Difficulty	0.07556	2.89204	19	1.70000	0.39014	-2.2500
Ability	0.15268	0.46331	19	0.68067	0.15616	-1.91428
t = -0.15874, p = 0.87564 (higher significant level) = lower correlation coefficient.						

Before implementing the second part of our system, let us first analyze a real learner information database, Table 7, from Photoshop example in which learners used learning management system. In this table, there are 296 learners, three self tests, and one final examination. Learners receive one of five grades, either Excellent (*A*), Very Good (*B*), Good (*C*), Fair (*D*) or Poor (*F*) on each of the components of the course.

Table 7. Learner information table.

U	Q1	Q2	Q3	Final	Freq
1	B	C	C	C	2
2	B	A	B	A	5
3	A	C	B	C	5
4	F	C	D	D	5
5	B	A	A	A	2
6	A	A	A	B	4
7	A	B	A	C	5
8	F	D	F	D	8
9	D	F	F	F	10
10	F	F	D	F	17
11	C	D	F	D	6
12	C	F	F	F	3
13	F	D	F	F	18
14	A	B	A	B	10
15	A	B	A	F	13
16	C	F	F	F	12
17	C	F	F	F	20
18	C	D	D	D	16
19	F	C	F	F	17
20	D	D	F	F	13
21	F	D	F	D	33
22	A	D	A	C	10
23	A	F	A	B	14
24	B	F	A	A	2
25	F	F	D	D	15
26	A	F	B	C	12
27	B	D	B	A	17
28	B	D	C	C	2

By taking into account, at the beginning of this algorithm, we reduce this table by using reduction algorithm, As in Table 1, the probability values of decision classes are $P(X_A)$= 0.087, $P(X_B)$= 0.094, $P(X_C)$= 0.121, $P(X_D)$= 0.280 and $P(X_F)$= 0.415.
Where:

$X_t=X_F,\ X_c=\{X_A, X_B, X_C, X_D\}.$
$X_f=\{u9,u10,u12,u13,u15,u16,u17,u19,u20\}$

The target parameter α is assumed to be 0.35; thus, β= -0.817. The target concept is "why students are fail in the final test". In the other word, we are only interested in what learning activity cause learners to fail the course; we use the poor concept in our discussion. By using equation 7, the following characteristics are obtained as shown in Table 8.

Table 8. Approximation regions.

Q_i	$POS^{0.35}(X_F)$	$NEG^{0.35}(X_F)$	$BND^{0.35}(X_F)$	η
Q1	$\{E_D\}$	$\{E_B\}$	$\{E_F,E_A,E_C\}$	0.179
Q2	$\{E_B,E_F,E_C\}$	$\{E_A\}$	$\{E_D\}$	0.584
Q3	θ	$\{E_C,E_B\}$	$\{E_A,E_F,E_D\}$	0.145

By comparing the discriminant indices of all tests, we find that *Q2*= 0.584 best determines the membership in X_F. Thus, we obtain the first rule:

R1: If Q2=F then Final=F confidence=59%.

To find the new domain, we first delete all elements that are not needed from Table 7, these elements are a positive and negative regions for *Q2*:
$\{E_B,E_F,E_C\} \cup \{E_A\}$.
Therefore, the new set of elements is:
$\{E_D\}=\{u8,u11,u13,u18,u20,u21,u22,u27,u28\}$.
The same procedure is applied to find the remaining rules:

R2: If Q1=F and Q2=D then Final=F Conf=100%.

R3:If Q3=F and Q2=D and Q1=F then Final=F on f=53%.

7. Conclusions

This study proposes a comprehensive recommendation system based on evaluation of the learner's ability and the learner's performance using PCM and modified BRS, respectively. First, PLR-PCM agent can estimate the abilities of the online learners and recommend appropriate learning resources to the learners based on PCM, which provides personalized web-based learning according to difficulty of the learning resources visited by the learners and their explicit and implicit responses. Second, BRS-Recommender agent can recommend a group of learners to guide them in their learning based on modified BRS, which specifies the areas they should focus on according to how they fit the decision rules.

The proposed system can prevent the learner from becoming lost in the learning resources, by providing personalized learning guidance, filtering out inappropriate learning resources to reduce cognitive loading, and providing a fine learning diagnosis based on an individual's user profile. Experimental results show that the proposed system can provide personalized learning resource recommendations with a high degree of correlation between the learner's ability and the recommended learning resource.

Additionally, it can also provide appropriate recommendation rules to the learner, resulting in increased learning efficiency and performance.

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