Image Zooming Technique Based on the Split Bregman Iteration with Fractional Order Variation Regularization

Liping Wang, Shangbo Zhou, and Awudu Karim
College of Computer Science, Chongqing University, China

Abstract: *It is always a challenging work to develop an accurate and effective method to reconstruct a degraded image. In this paper, the nonlocal variation Fractional Total Variation (FTV) regularization technique for image zooming is investigated. To enhance edges, yet preserve textures, fractional order calculus based image zooming method is proposed, which can deal well with fine structures like textures. To solve the nonlinear Euler-Lagrange equation associated with the nonlocal variation FTV regularization model, we propose a nonlocal total variation method for image zooming based on the split Bregman iteration. Enlarging and de-noising experimental results show that the proposed method has effectiveness and reliability by comparing to some methods mentioned in the paper.*

Keywords: *Image zooming, total variation, split bregman iteration, fractional order.*

Received December 26, 2014; accepted June 1, 2015

1. Introduction

Image zooming technique is increasingly popular nowadays, because it is important in image application fields such as medical imaging [37], electronic publishing, satellite images, images found on web [2, 12, 24], license plate identification and face recognition system.

A variety of zooming methods were proposed in the past few decades. The simplest method for enlarging an image is pixel replication, which is easy to implement. But the method may cause the processed image blurring and blocky effects. Bilinear Interpolation (BLI) [25] and Bicubic Interpolation (BCI) [21], which are both with superior accuracy than pixel replication, were proposed, due to BLI and BCI use polynomials for up-sampling image. However, these linear interpolation methods are easy to complement, but would degrade the zoomed quality. Whereas, the higher order interpolation method can gives better performances, but requires much computation. The ordinary drawbacks of the above methods cause desired image jagged or synthetic edges or low-quality edge blurring. Thus, finding a method that can eliminate blocky effects and look natural in processed image is a focus of current research.

In recent years, methods based on Partial Differential Equations (PDEs) have been proposed and have shown better performance than previous methods. They include anisotropic diffusion equations [8, 10, 28, 33], total variation models [32] and curve evolution equations [22]. One of the most famous techniques, which can well solve the image reconstruct problem, is the Total Variation (TV) regularization method proposed by Rudin, Osher, and Fatemi (ROF) model, which has the following form:

$$\min_u |\tilde{N}u| + \frac{\lambda}{2}\|u - f\|_2^2 \tag{1}$$

Where f stands for the observed data, u represents the desired data, $|\cdot|$ denotes the l_1 norm of its variation, $\lambda>0$ is a regularization parameter. The ROF model can well protect the edge information. However, staircase effects are generated when enlarging the image with classical TV norm.

In order to remove staircase effects, the authors of [6] exploited a higher order nonlinear partial differential equation to smooth the image. In literatures [7, 18, 27, 36], the authors have proposed second-order or fourth-order telegraph-diffusion equations for noise removal, which adding a second time derivative of the image in PDEs, these methods can effectively eliminate the diffusion effect to some degree. These methods mentioned above are based on local image operators, which preserve edges and smooth regions and de-noise very well, but they cannot deal well with fine structures like texture, because texture is not local in nature [20].

Recently, a lot of nonlocal methods have been proposed for image processing. Gilboa and Osher [15] proposed a Nonlocal H^1 (NL-H^1) regularization energy variational formulation by embedding the nonlocal means. Subsequently, Gilboa and Osher [16] presented a variational model, which depends on Nonlocal Total Variation (NL-TV). Yang *et al.* [35] proposed a split Bregman algorithm for the NL-TV regularization image zooming. Mathieu *et al.* [26] proposed an edge detection method by introducing the fractional differentiation. Bai and Feng [4] proposed a new method of fractional order anisotropic diffusion equations for noise removal. A Fractional order TV (FTV) regularization functional for image super-resolution was proposed in literature [31], which integrated traditional TV, fractional TV and data fidelity term.

Inspired by the ideas of [31, 35], in this paper, we proposed a method to use the FTV regularization model to up-sample images. To solve the FTV variational minimization problem in the discrete case, we proposed a method based on split Bregman iteration algorithm and FTV regularization for image zooming. The split Bregman method has some advantages: Fast computational speed; relatively small memory footprint and easy to code. Numerical experiments show that the proposed Bregmanized nonlocal total variation-fractional total variation regularization algorithm can recover more details of the image than other methods.

The organization of the rest of the paper is as follows. In section 2, we give a description of related work. The proposed model and its discretization are introduced in section 3. Section 4 is devoted to implementation details of numerical experiments. Finally, some conclusions are given in section 5.

2. Related Work

The aim of image zooming is to estimate the desired image from a given degraded image. Because of the ill-posed nature of super resolution, Equation 1 is not a unique model. To find an effective method of image zooming, lots of authors try to presents many algorithms [2, 23, 34, 35, 37]. In recent years, many varieties of total variation were presented based on non-local notion [14, 23, 29, 30, 34, 35]. Those algorithms were proposed based on non-local notion, which can deal fine with structures such as texture because texture is a non-local feature. Pu *et al.* [29, 30] discussed the capabilities of the fractional differential approach for enhancing texture features of two-dimensional digital images. Bai and Feng [4] introduced a new class of fractional order anisotropic diffusion equations for image de-noising. The following section will reviews the non-local operator of fractional calculus and the split Bregman iteration algorithm.

2.1. The Non-local Operator of Fractional Calculus

Fractional calculus is proposed with respect to the traditional integer order calculus, and fractional calculus can be seen as the generalization of the integer order calculus. We can find more than one fractional order derivative exist in literatures, such as definition of Rieman-Liouville (R-L) and Grünwald-Letnikov (G-L). However, we use the frequency domain definition in this paper because it has less computation. For any function $f(t) \in L^2(R)$, the Fourier transform of it is:

$$\hat{f}(\omega) = \int f(t) exp(-j\omega t) dt \tag{2}$$

Where $j = \sqrt{-1}$ is the imaginary unit. The equivalent formulation of the n^{th} derivative in the frequency domain is

$$D^n f(t) \leftrightarrow (j\omega)^n \hat{f}(\omega) \tag{3}$$

Where "$\leftrightarrow$" stands for the Fourier transform pair. It is meaningful for any number n at the right-hand side of Equation 3. So, the fractional order derivative of the function $f(t)$ of order v is defined as:

$$D^v f(t) = F^{-1}((j\omega)^v \hat{f}(\omega)), \quad v > 0 \tag{4}$$

Similarly, for any $f(x, y) \in L^2(R^2)$, the corresponding 2-D fractional order derivatives of it have the following forms [4]:

$$D_x^v f(x, y) = F^{-1}((j\omega_1)^v \hat{f}(\omega_1, \omega_2)) \tag{5}$$

$$D_y^v f(x, y) = F^{-1}((j\omega_2)^v \hat{f}(\omega_1, \omega_2)) \tag{6}$$

So, fractional order gradient is defined as:

$$Grad_{ftv} f(x, y) = D_x^v f(x, y) i + D_y^v f(x, y) j \tag{7}$$

In order to write simply, replace gradient symbol $Grad_{ftv}$ with G_{ftv}. And the module of fractional order gradient according to Equation 7 can be written as:

$$|G_{ftv}| = \sqrt{(D_x^v f)^2 + (D_y^v f)^2} \tag{8}$$

Assuming the vector field $A(x, y)$ is given by:

$$A(x, y) = P(x, y) i + Q(x, y) j \tag{9}$$

Where $P(x, y)$ and $Q(x, y)$ are two components of $A(x, y)$ which having fractional v order continuous derivatives, so $D_x^v P + D_y^v Q$ be called fractional order divergence of vector field $A(x, y)$, written as $divA$, that is:

$$div_{ftv} A = D_x^v P + D_y^v Q \tag{10}$$

The fractional Laplacian is defined by:

$$\Delta^v u = div_{ftv}[G_{ftv}] = \tilde{N}_x^v \tilde{N}_x^v u + \tilde{N}_y^v \tilde{N}_y^v u \tag{11}$$

We consider the following functional defined in the space of continuous images over a support of Ω, then the fractional TV regularization defined by:

$$E(u) = \int_{\Omega} f(|D^v u|) d\Omega \tag{12}$$

The corresponding Euler-Lagrange equation can be written as:

$$D_x^{v^*}(c(|D_x^v u|^2)D_x^v u) + D_y^{v^*}(c(|D_y^v u|^2)D_y^v u) = 0 \tag{13}$$

Where $D_x^{v^*}$ and $D_y^{v^*}$ are adjoints of $D_x^v u$ and $D_y^v u$, respectively. $c(\bullet)$ is a diffusion coefficient given by:

$$c(s) = \frac{f'(\sqrt{s})}{s} \tag{14}$$

2.2. The Split Bregman Iteration Algorithm

Bregman iteration is a new method of image restoration based on the ROF model in recent years. Goldstein and Osher [17] presented a fast computation split Bregman iterative method using the technique of separation variables.

- *Definition 1*: Subdifferential for arbitrary convex functional J: $\chi \to R$, it's subdifferential $\partial J(u)$ at u be defined as [35]:

$$\partial J(u) := \left\{ p \in \chi^* \mid J(v) \geq J(u) + \langle p, v-u \rangle . \forall v \in \chi \right\} \tag{15}$$

- *Definition 2*: Bregman distance assuming J is a convex functional, $\partial E(v)$ is subdifferential at v of functional J, $p \in \partial J(v)$, then Bregman distance of u and v with regard to functional J is defined as [35]:

$$D_J^p(u,v) := J(u) - J(v) - \langle p, v-u \rangle \tag{16}$$

The split Bregman iteration is used for the l_1-regularized of the object function. Equation 1 can be rewrite equivalently as follows:

$$\min_u |\phi(u)| + H(u,f) \tag{17}$$

Let d denotes the function of u, $d=\phi(u)$. Then we get an effective solution of the Equation 17 using the split Bregman iteration method:

$$\begin{aligned}(u^{k+1}, d^{k+1}) &= \arg\min_{u,d} D_E^p(u,u^k,d,d^k) + \frac{\lambda}{2}\left\|d^k - \phi(u^k)\right\|_2^2 \\ &= \arg\min_{u,d} E(u^k,d^k) - \langle p_u^k, u-u^k \rangle - \langle p_d^k, d-d^k \rangle + \frac{\lambda}{2}\left\|d^k - \phi(u^k)\right\|_2^2\end{aligned} \tag{18}$$

$$p_u^{k+1} = p_u^k - \lambda(\nabla\phi)^T(\phi u^{k+1} - d^{k+1}) \tag{19}$$

$$p_d^{k+1} = p_d^k - \lambda(d^{k+1} - \phi u^{k+1}) \tag{20}$$

We can get the following simplified format by the equivalence of Bregman iterative first form and the second form.

$$(u^{k+1}, d^{k+1}) = \arg\min_{u,d} \|d^k\|_1 + H(u^k,f) + \frac{\lambda}{2}\left\|d^k - \phi(u^k) - b^k\right\|_2^2 \tag{21}$$

$$b^{k+1} = b^k - (d^{k+1} - \phi u^{k+1}) \tag{22}$$

For Equation 21, we can perform this minimization efficiently by iteratively minimizing with respect to u and d separately. The two steps we must perform are:

- *Step 1.* $u^{k+1} = \arg\min_u H(u^k,f) + \frac{\lambda}{2}\left\|d^k - \phi(u^k) - b^k\right\|_2^2$
- *Step 2.* $d^{k+1} = \arg\min_d \|d^k\|_1 + \frac{\lambda}{2}\left\|d^k - \phi(u^{k+1}) - b^k\right\|_2^2$

There are many methods for solving steps 1 and 2, the specific discussion can be found in literature [17].

3. The Proposed Image Zooming Method

In this section, we firstly give a description of the proposed fractional total variation image zooming method and discretization of the proposed method.

We know that the regularization process is to smoothing an image using the curvature in the ROF model, and the higher curvature of the image is, the stronger the regularization is. Yet, the traditional total variation norm constraints limit details in image with higher curvature [3, 9]. We propose the image zooming method based on fractional order TV using the split Bregman iteration because fractional calculus can enhance the high frequency while preserving the low frequency information in image. The proposed model is as follows:

$$\min_u |\nabla_x^v u| + |\nabla_y^v u| + \frac{\mu}{2}\|u-f\|_2^2 \tag{23}$$

Where $\mu>0$ is a regularization parameter.

According to the split Bregman iteration algorithm, we rewrite the Equation 23 as follows:

$$\begin{cases} u^{k+1} = \arg\min_u \frac{u}{2}\|u-f\|^2 + \frac{\lambda}{2}\|d_x^k - \tilde{N}_x^v u - b_x^k\|^2 \\ \qquad\qquad + \frac{\lambda}{2}\|d_y^k - \tilde{N}_y^v u - b_y^k\|^2 \\ d^{k+1} = \arg\min_d |d^k| + \frac{\lambda}{2}\|d^k - \tilde{N}^v u - b^k\|^2 \\ b^{k+1} = b^k + \tilde{N}^v u^{k+1} - d^{k+1} \end{cases} \tag{24}$$

The Euler-Lagrange equation for the solution u^{k+1} in Equation 23 are given by:

$$\mu(u^{k+1} - f) - \lambda\nabla_x^v(d_x^k - b_x^k) - \lambda\nabla_y^v(d_y^k - b_y^k) + \lambda\Delta^v u^{k+1} = 0 \tag{25}$$

Which leads to:

$$u^{k+1} = \frac{1}{\mu + \lambda\Delta^v}[\mu f + \lambda\nabla^v(d_x^k - b_x^k) + \lambda\nabla^v(d_y^k - b_y^k)] \tag{26}$$

The solution of d^{k+1} is corresponding to a shrinkage operator:

$$d^{k+1} = shrink\left(\nabla^v u^{k+1} + b^k\right) \tag{27}$$

Where operator $shrink(x,y)$ equals to the equation $sgn(x)*max(|x|-y, 0)$.

Finally, we update the Bregman variable simply by renewing b^{k+1},

$$b^{k+1} = b^k + \nabla^v u^{k+1} - d^{k+1} \tag{28}$$

The flow chart of this proposed method is given as follows:

- *Initialization*: choose $u^0=f$, $d^0=b^0=0$, set k=0, set convergence precision *tol* and inner loop number *N*.

$$
\begin{aligned}
&\textit{while} \quad u^k - u^{k-1} \\
&\quad \textit{for} \quad n = 1 \ \textit{to} \\
&\qquad u^{k+1} \leftarrow \arg\min_u \frac{u}{2} \| u - f \|^2 + \frac{\lambda}{2} \| d_x^k - \nabla_x^v u - b_x^k \|^2 + \frac{\lambda}{2} \| d_y^k - \nabla_y^v u - b_y^k \|^2 \\
&\qquad d^{k+1} \leftarrow \arg\min_d | d^k | + \frac{\lambda}{2} \| d^k - \nabla^v u - b^k \|^2 \\
&\quad \textit{end} \\
&\quad b^{k+1} = b^k + \nabla^v u^{k+1} - d^{k+1} \\
&\textit{end while}
\end{aligned}
$$

4. Numerical Experiments and Analysis

In this section, various experiments are implemented on white Gaussian noise under MATLAB 2013a for validating the performance and effectiveness of the proposed algorithm. The experimental results of methods based on fractional TV are compared with some well-known algorithms, such as: BLI [25], BCI [21], the classical ROF model-based image restoration method [32], the fourth-order model PDE image restoration method [13], and NL-TV [23]. The experiments environment is: Microsoft Windows 7 operating system, Matlab 2013a and PC with an Intel Core(TM) i3-2120 CPU at 3.30GHz and 8.00GB of memory.

Firstly, we zoomed three group images to test capability of the proposed method in the image amplification. Secondly, in order to measure the quality of numerical results of the proposed method, three concepts are introduced: Peak Signal-to-Noise Ratio (PSNR), the Mean Squared Error (MSE), and Mean Structural Similarity Index Measurement (MSSIM), respectively, defined at discrete level by:

$$PSNR = 10 log_{10} \frac{255^2}{MSE} \tag{29}$$

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[I(i,j) - \hat{I}(i,j) \right]^2 \tag{30}$$

$$MSSIM = \frac{1}{L} \sum_{i=1}^{L} \left[l(I,\hat{I}) \right]^a \cdot \left[c(I,\hat{I}) \right]^b \cdot \left[s(I,\hat{I}) \right]^c \tag{31}$$

Where, $m \times n$ is the size of the image, I and $\hat{I}$ are the original and restored image, respectively, L is the number of blocks of the image. $l(\cdot)$, $c(\cdot)$, $s(\cdot)$ are luminance, contrast, structure comparison functions respectively. a>0, b>0, c>0 are weighted parameters that used to regulate the relative importance of the three components. The stopping criterion that checking the maximum variation between u^{k+1} and u^k is less than 10^{-3}. Higher *MSSIM* indicates more similar structure between the reconstructed image and the original image. The lower *MSSIM* is, the more precise the reconstructed image is.

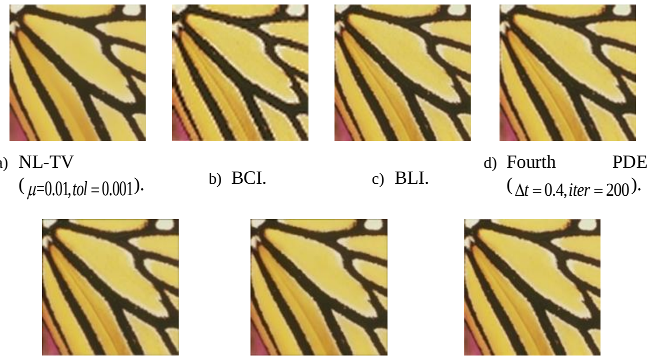
In order to test the effectiveness of the proposed method, we do some experiments on gray-scale images: “bara”, “cameraman” and RGB color images: “butterfly”, “parrot”. We firstly state the effectiveness of the proposed method with images: “bara”, “cameraman”, “butterfly” and “parrot” enlarging 4 times, respectively. Secondly, we magnify the part of images “butterfly” and “parrot”. Finally, adding additive white Gaussian noise of σ=25 on images “boat”, we compare performance of the proposed method with other methods.

Capability of different image restoration algorithms: *PSNR*, *MSE* and *MSSIM* results on five noisy images (additive Gaussian noise, variance is 25, mean value is zero) for 4×magnification. For each image, we have three rows. The upper row is *PSNR*, the middle row is *MSE*, the under row is *MSSIM*.

The compared results of image zooming experiments are shown as in Figures 1, 2 and 3. The BCI method brings the most unsatisfied zooming images with blurred edge.

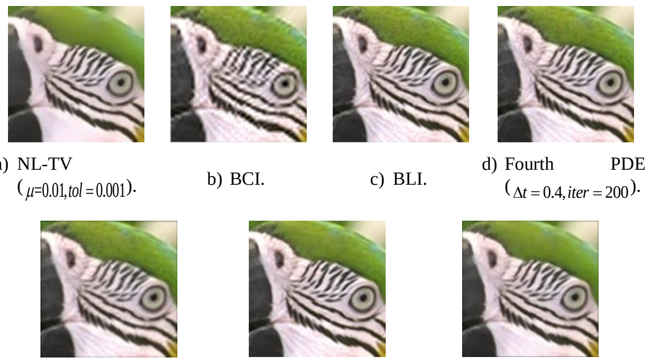
Figure 1. Comparison(4×)of zooming results on images by different methods (from top to button: Methods are BCI [21], BLI [25], fourth PDE [13] ($\Delta t = 0.4, iter = 200$), ROF [32] ($\lambda = 0.05, tol = 0.001$), NL-TV [32] (μ=0.01, $tol = 0.001$), NL-BF [1], proposed ($\lambda = 0.05, tol = 0.001$)).

The NL-TV and ROF methods are able to effectively suppress jaggy artifacts along edges, but it generate obvious blur. The fourth PDE and BLI methods can effectively suppress blur, but omit many fine details. The proposed method can obtain better result than the previous methods, which is most accurate to the ground truth. It can prevent artifacts and produce zooming images with sharper edges and finer details. This is mainly due to fractional order total variation, which can better handle non-local details than integer order total variation. Therefore, the proposed method can availably rebuild more reliable zooming images.



a) NL-TV (μ=0.01, $tol=0.001$). b) BCI. c) BLI. d) Fourth PDE ($\Delta t=0.4, iter=200$). e) ROF ($\lambda=0.05, tol=0.001$). f) NL-BF. g) Proposed ($\lambda=0.05, tol=0.001$).

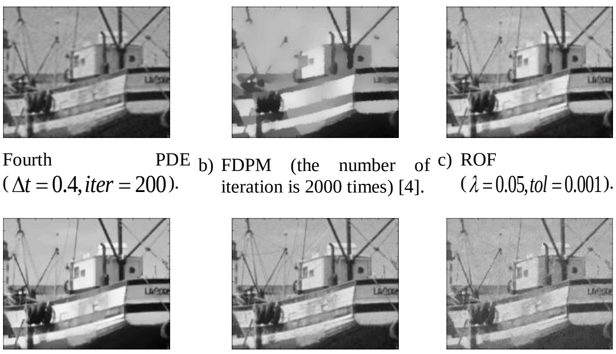
Figure 2. Portion of zooming results on image butterfly by different methods.



a) NL-TV (μ=0.01, $tol=0.001$). b) BCI. c) BLI. d) Fourth PDE ($\Delta t=0.4, iter=200$). e) ROF ($\lambda=0.05, tol=0.001$). f) NL-BF. g) Proposed ($\lambda=0.05, tol=0.001$).

Figure 3. Portion of zooming results on image parrot by different methods.

Figure 4 illustrates the results of de-noising on image boat.



a) Fourth PDE ($\Delta t=0.4, iter=200$). b) FDPM (the number of iteration is 2000 times) [4]. c) ROF ($\lambda=0.05, tol=0.001$). d) NL-TV (μ=0.01, $tol=0.001$). e) Proposed (μ=0.01, $tol=0.001$). f) NL-BF [1].

Figure 4. Performance of de-noising (Gaussian white noise, variance is 25, mean value is zero) by different methods.

We can see that the result of the fourth PDE method is the worst, where there are a lot of noises. Fractional Diffusion Perona-Malik (FDPM) and NL-TV methods can effectively handle noise, but many details are smoothed. The ROF method can preserve edges, meanwhile smoothed some noises, but it not as good as the result of the proposed method in preserve edges and in visual effect.

In Table 1, the quantitative evaluation is given for different restoration algorithms for the Bara, Cameraman, Lena, Boat and Dollar images using the index of *PSNR*, *MSE* and *MSSIM* under the same noise intensity. From table, it is found that the value of *PSNR*, *MSE* and *MSSIM* of the proposed method are almost best. So the proposed method is effectively for image de-noising.

Table 1. Capability of different image restoration algorithms.

Images		Methods				
		Fourth PDE	**FDPM [4]**	**ROF**	**NL-TV**	**Proposed**
Bara	**PSNR**	28.7669	24.8577	24.4845	24.9114	29.2213
	MSE	86.3755	212.48	231.5435	209.8661	86.0100
	MSSIM	0.6464	0.7870	0.8353	0.8567	0.8360
Cameraman	**PSNR**	28.7828	26.7491	27.4706	29.2041	30.0463
	MSE	86.0600	137.46	116.4181	78.1037	64.3361
	MSSIM	0.5359	0.7213	0.8931	0.9228	0.9298
Lena	**PSNR**	28.7592	27.1389	28.3576	26.6458	29.5955
	MSE	86.7284	125.66	94.9110	140.7659	85.8452
	MSSIM	0.3320	0.7741	0.9064	0.7934	0.9178
Boat	**PSNR**	28.7709	26.9312	28.4389	25.7276	29.4995
	MSE	86.2950	131.81	93.1507	173.9074	82.2943
	MSSIM	0.3695	0.7878	0.9005	0.7383	0.9177
Dollar	**PSNR**	28.7427	23.0232	20.7987	22.9944	28.3546
	MSE	86.8585	324.16	541.0186	326.3161	87.1957
	MSSIM	0.7535	0.8339	0.8732	0.8712	0.9713

5. Conclusions

In this paper, we propose an image zooming method combined fractional order total variation and the split Bregman iteration. The proposed method make use of the characteristic of fractional order calculus: Fractional order calculus can enhance the high frequency information (such as edges), and preserve the low frequency information (such as texture, details). Our model can magnifies the resolution of a low resolved image and is able to rebuild when it degraded by noise, blur, and down-sampling. We solve the nonlinear Euler-Lagrange equation by using the split Bregman iteration, which has been demonstrated to be an efficient and accurate algorithm related in section 2.2. The good performance of the proposed method has been tested for image degraded with Gaussian white noise and up-sampling. Quantitative experiment results demonstrate that the proposed method does better than other state-of-the-art methods. In our future work, we may utilize other energy functional to reconstruct image using the split Bregman iteration.

Acknowledgments

This work was supported by The Fundamental Research Funds for the Central Universities (No.106112014CDJZR188801), National Natural Science Foundation of China under Grant (No.

60873200), and Frontier and Application Foundation Research Program of CQ CSTC (No.cstc2014jcyjA40037).The authors also thank sincerely the anonymous referee for his or her valuable comments, which helped in improving substantially the manuscript.

References

[1] Anh D., "Iterative Bilateral Filter and Non-Local Mean," *International Journal of Computer Applications*, vol. 106, no. 11, pp. 33-38, 2014.

[2] Almira J. and Romero A., "Image Zooming Based on Sampling Theorems," *Materials Mathematics*, vol. 11, no. 1, pp. 1-22, 2011.

[3] Aubert G. and Kornprobst P., *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations*, Springer-Verlag, 2002.

[4] Bai J. and Feng X.,"Fractional-Order Anisotropic Diffusion for Image Denoising," *IEEE Transactions on Image Processing*, vol. 16, no. 10, pp. 2492-2502, 2007.

[5] Bregman L., "The Relaxation Method of Finding the Common Points of Convex Sets and its Application to the Solution of Problems in Convex Optimization," *USSR Computational Mathematics and Mathematical Physics*, vol. 7, no. 3, pp. 200-217, 1967.

[6] Bruni C., Bruni R., and Santis A., "Global Optimal Image Reconstruction from Blurred Noisy Data by a Bayesian Approach," *Journal of Optimal Theory and Applications*, vol. 115, no. 1, pp. 67-96, 2002.

[7] Cao Y., Yin J., Liu Q., and Li M., "A Class of Nonlinear Parabolic-Hyperbolic Equations Applied to Image Restoration," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 253-261, 2010.

[8] Catte F., Lions P., Morel J., and Coll T., "Image Selective Smoothing and Edge Detection by Nonlinear Diffusion," *SIAM Journal on Numerical Analysis*, vol. 29, no. 1, pp. 182-193, 1992.

[9] Chan T. and Shen J., *Image Processing and Analysis-Variational, PDE, Wavelet, and Stochastic Methods*, SIAM Philadelphia, 2005.

[10] Chen Q., Montesinos P., Sun Q., and Xia D., "Ramp Preserving Perona-Malik Model," *Signal Process*, vol. 90, no. 6, pp. 1963-1975, 2010.

[11] Chung F., *Spectral Graph Theory*, American Mathmatical Science, 1997.

[12] Combs T. and Bederson B., "Does Zooming Improve Image Browsing?," *in Proceedings of 4th ACM conference on Digital libraries*, California, pp. 130-137, 1999.

[13] Gao R., Song J., and Tai X., "Image Zooming Algorithm Based on Partial Differential Equations Technique," *International Journal of Numerical Analysis and Modeling*, vol. 6, no. 2, pp. 284-292, 2009.

[14] Gilboa G. and Osher S., "Nonlocal Operators with Applications to Image Processing," *Society for Industrial and Applied Mathematics Journal Multiscale Modeling and Simulation*, vol. 7, no. 3, pp. 1005-1028, 2008.

[15] Gilboa G. and Osher S., "Nonlocal Linear Image Regularization and Supervised Segmentation," *Multiscale Modeling and Simulation*, vol. 6, no. 2, pp. 595-630, 2007.

[16] Gilboa G. and Osher S., "Nonlocal Operators with Applications to Image Processing," *Multiscale Modeling and Simulation*, vol. 7, no. 3, pp. 1005-1028, 2008.

[17] Goldstein T. and Osher S., "The Split Bregman Method for L1 Regularized Problems," *Society for Industrial and Applied Mathematics Journal Imaging Sciences*, vol. 2, no. 2, pp. 323-343, 2009.

[18] Ilyevsky A. and Turkel E., "Stopping Criteria for Anisotropic PDEs in Image Processing," *Journal of Scientific Computing*, vol. 45, no. 1, pp. 333-347, 2010.

[19] Jia R., Zhao H., and Zhao W., "Convergence Analysis of the Bregman Method for the Variational Model of Image Denoising," *Applied and Computational Harmonic Analysis*, vol. 27, no. 3, pp. 367-379, 2009.

[20] Jung M., Bresson X., Chan T., and Vese L., "Nonlocal Mumford-Shah Regularizers for Color Image Restoration," *IEEE Transactions on Image Processing*, vol. 20, no. 6, pp. 1583-1598, 2011.

[21] Keys R., "Cubic Convolution Interpolation for Digital Image Processing," *IEEE Transactions on Acoustics Speech Signal Process*, vol. 26, no. 6, pp. 508-517, 1978.

[22] Kimia B., Tannenbaum A., and Zucker S., "On the Evolution of Curves via a Function of Curvature," *Journal of Mathematical Analysis and Applications*, vol. 163, no. 2, pp. 438-58, 1992.

[23] Li L., Xie Y., Hu W., and Zhang W., "Single Image Super-Resolution Using Combined Total Variation Regularization by Split Bregman Iteration," *Neurocomputing*, vol. 142, pp. 551-560, 2014.

[24] Lukac R. and Plataniotis K., "Bayer Pattern Based Digital Zooming Approach," *in Proceedings of 2004 International Symposium on Circuits and Systems*, Vancouver, pp. 253-258, 2004.

[25] Maeland E., "On the Comparison of Interpolation Methods," *IEEE Transactions on Medical Imaging*, vol. 7, no. 3, pp. 213-217, 1988.

[26] Mathieu B., Melchior P., Oustaloup A., and Ceyral C., "Fractional Differentiation for Edge

Detection," *Signal Processing*, vol. 11, no. 83, pp. 2421-2432, 2003.

[27] Rahman M., PK M., and Uddin M., "Optimum Threshold Parameter Estimation of Bidimensional Empirical Mode Decomposition Using Fisher Discriminant Analysis for Speckle Noise Reduction," *The International Arab Journal of Information Technology*, vol. 12, no. 5, pp. 456-464, 2015.

[28] Perona P. and Malik J., "Scale Space and Edge Detection Using Anisotropic Diffusion," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629-639, 1990.

[29] Yi-fei P., "Application of Fractional Differential Approach to Digital Image Processing," *Journal of Sichuan university*, vol. 39, no. 3, pp. 124-133, 2007.

[30] Pu Y. and Wang W., "Fractional Differential Masks of Digital Image and Their Numerical Implementation Algorithms," *ACTA Automatic Sinica*, vol. 33, no. 11, pp. 1128-1135, 2007.

[31] Ren Z., He C., and Zhang Q., "Fractional Order Total Variation Regularization for Image Super-Resolution," *Signal Processing*, vol. 93, no. 9, pp. 2408-2421, 2013.

[32] Rudin L., Osher S., and Fatemi E., "Nonlinear Total Variation Based Noise Removal Algorithms," *Physica D: Nonlinear Phenomena*, vol. 60, no. 1-4, pp. 259-268, 1992.

[33] Wei G., "Generalized Perona-Malik Equation for Image Restoration," *IEEE Signal processing letters*, vol. 6, no. 1, pp. 165-167, 1999.

[34] Xiang F. and Wang Z., "Split Bregman Iteration Solution for Sparse Optimization in Image Restoration," *International Journal for Light and Electron Optics*, vol. 125, no. 19, pp. 5635-5640, 2014.

[35] Yang Y., Wu T., and Pang Z., "Image-Zooming Technique Based on Bregmanized Nonlocal Total Variation Regularization," *Optical Engineering*, vol. 50, no. 9, pp. 1-10, 2011.

[36] Zeng W., Lu X., and Tan X., "Non-Linear F ourth-Order Telegraph-Diffusion Equation for Noise Removal," *IET Image Processing*, vol. 7, no. 4, pp. 335-342, 2013.

[37] Zhu X., Lei W., and Zhang S., "Image Zooming Using c and e Model and its Performance Evaluation," *in Proceedings of Intelligent Control and Information Processing*, pp. 262-265, 2010.

Liping Wang received his BSc in Information Management and Information System from Zhengzhou university, Zhengzhou, China, in 2004. He worked as a teacher at college of Shengda economics trade and management of Zhengzhou university from 2004-2007.Received his master in technology of computer application from Chongqing university of posts and telecommunications, Chongqing, in 2010.he is currently working toward her PhD in college of computer science at Chongqing university. His research interests include image restoration, pattern recognition, and inverse problems.

Shangbo Zhou received the BSc degree from Gangxi National College in 1985, the MSc degree from Sichuan University in 1991, both in math, and PhD degree in circuit and system from Electron Science and Technology University. From1991 to 2000, he was with the Chongqing aerospace electronic and mechanical technology design research institute. Since 2003, he has been with the college of computer science and engineering of Chongqing University, where he is now a professor. His current research interests include artificial neural networks, physical engineering simulation, image processing and nonlinear dynamics.

Awudu Karim was born in 1976, at Dunkwa in the central region of Ghana. He graduated from a 3-year college of education program in 1999 with a teaching certificate 'A', and became a professional junior high school science teacher with Ghana education service afterwards. He received his bachelor's degree in information technology from the University of Education, Winneba, in 2008. He worked briefly as ICT coordinator/instructor with the ministry of education. He received M.Eng. degree in computer science from Hunan University, P.R.C., in 2012. He was a member of IOT team of Hunan University headed by Dr. Zhang Xiaoming from 2011 to 2012. He is currently a Ph.D student in computer science, Chongqing University, P.R.C.