

# Universal Forgery Attack on a Strong Designated Verifier Signature Scheme

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**Abstract:** Based on the bilinear Diffie-Hellman assumption, in 2009, Kang *et al.* proposed an identity-based strong Designated Verifier Signature (DVS) scheme which only allows the intended verifier to verify the signature. Besides, the designated verifier is not capable of transferring the conviction to any third party. Their scheme was proved secure in the random oracle model. In this paper, however, we will demonstrate that their scheme is still vulnerable to the universal forgery attack for arbitrarily chosen messages. Moreover, an efficient and provably secure improvement to eliminate the security weakness is presented.

**Keywords:** Universal forgery, identity-based, designated verifier, digital signature, bilinear pairing.

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## 1. Introduction

In 1996, Jakobsson *et al.* [2] proposed the so-called DVS scheme. In a DVS scheme, anyone can verify the corresponding signature with signer's public key. However, only the intended verifier will be convinced of the signer's identity. Moreover, the designated verifier cannot transfer the conviction to any third party, as he also, has the ability to compute a valid DVS intended for himself. Saeednia *et al.* [7] further proposed a Strong Designated Verifier Signature (SDVS) scheme by combining the designated verifier's private key with the signature verification process, so that only the designated verifier can validate the signature. However, Lee and Chang [6] demonstrated that Saeednia *et al.*'s scheme could not fulfill the property of signer ambiguity in case that signer's private key is accidentally compromised.

Considering the identity-based systems, Susilo *et al.* [8] addressed the first identity-based SDVS scheme from bilinear pairings. Since then, several related works [1, 4, 9] have been proposed. Recently, Kang *et al.* [3] proposed an identity-based SDVS scheme which has not only lower computational costs, but also, shorter signature length. The security of their scheme is formally proved secure in the random oracle model. Yet, in this paper, we will show that their scheme is still vulnerable to the universal forgery attack for arbitrarily chosen messages. Then an efficient countermeasure to resist such an attack without increasing much computational costs is given.

The rest of this paper is organized as follows: section 2 briefly reviews Kang *et al.*'s scheme. We demonstrate the universal forgery attack on their scheme in section 3. An improvement to resist the attack is proposed in section 4. Finally, a conclusion is made in section 5.

## 2. Review of Kang *et al.*'s Scheme

In this section, we first define used notations in Table 1 and then briefly review Kang *et al.*'s scheme.

Table 1. The used notations.

$Z_q$	Integers modulo $q$
$x \in Z_q$	Element $x$ in set $Z_q$
$x \in_R Z_q$	Element $x$ is a random integer in set $Z_q$
$x \leftarrow Z_q$	Sampling element $x$ uniformly in set $Z_q$
$ x $	Bit-length of integer $x$ , also, absolute value of $x$

- **Bilinear Pairing:** Let  $(G_1, +)$  and  $(G_2, \times)$  denote two groups of the same prime order  $q$  and  $e: G_1 \times G_1 \rightarrow G_2$  be a bilinear map which satisfies the following properties:

1. **Bilinearity:** For  $P, Q \in G_1$  and  $a, b \in Z_q$ ,  $e(aP, bQ) = e(P, Q)^{ab}$ .
2. **Non-Degeneracy:** There exists  $P, Q \in G_1$  such that  $e(P, Q) \neq 1$ .
3. **Computability:**  $e(P, Q)$  can be efficiently computed for  $P, Q \in G_1$ .

Kang *et al.*'s scheme is composed of five phases (Setup, KeyExtract, Sign, Verify, Transcript simulation) described as follows:

- **Setup:** The Private Key Generation center (PKG) chooses a master secret key  $s \in_R Z_q$ , computes the corresponding public key  $P_{TA} = sP$  and then selects two groups  $(G_1, +)$  and  $(G_2, \times)$  of the same prime order  $q$ . Let  $P$  be a generator of order  $q$  over  $G_1$ ,  $e: G_1 \times G_1 \rightarrow G_2$  a bilinear pairing,  $H: \{0, 1\}^* \rightarrow G_1$  and  $F: \{0, 1\}^* \rightarrow Z_q$  cryptographic hash functions [5]. The PKG announces public parameters  $params = \{P_{TA}, G_1, G_2, q, P, e, H, F\}$

- *KeyExtract*: Given an identity  $ID$ , the PKG computes the private key  $S_{ID} = sQ_{ID}$  where  $Q_{ID} = H(ID)$  is the corresponding public key. The private key is then sent to the user via a secure channel.
- *Sign*: Let Alice be a signer and Bob the designated verifier. For signing a message  $m$  intended for Bob, Alice chooses  $k \in_R Z_q$  to compute:

$$t = e(P, Q_B)^k \quad (1)$$

$$T = kP + F(m, t)S_A \quad (2)$$

$$\sigma = e(T, Q_B) \quad (3)$$

The SDVS for  $m$  is  $(t, \sigma)$ .

- *Verify*: Given  $(m, t, \sigma)$ , Bob verifies whether:

$$\sigma = t \cdot e(Q_A, S_B)^{F(m, t)} \quad (4)$$

If it holds, Bob is convinced that  $(t, \sigma)$  is a valid SDVS for  $m$ .

- *Transcript Simulation*: To generate another SDVS intended for himself, Bob first chooses  $k^* \in_R Z_q$  and then computes  $t^* = e(P, Q_B)^{k^*}$  and  $\sigma^* = t^* \cdot e(Q_A, S_B)^{F(m, t^*)}$ . The derived  $(t^*, \sigma^*)$  is another valid SDVS for  $m$ .

### 3. Universal Forgery Attack on Kang *et al.*'s Scheme

To launch the universal forgery attack on Kang *et al.*'s scheme for an arbitrarily chosen message  $m''$ , a malicious adversary first intercepts an SDVS intended for Bob, say  $(m, t, \sigma)$ , and then chooses  $t'' \in_R G_2$  to compute:

$$\sigma'' = t'' \cdot ((t^{-1} \sigma)^{F(m, t)^{-1}})^{F(m'', t'')} \quad (5)$$

The forged SDVS for  $m''$  is  $(t'', \sigma'')$ . We claim that  $(t'', \sigma'')$  will pass the signature verification, as the shared secret between Alice and Bob can be easily derived by computing:

$$e(Q_A, S_B) = (t^{-1} \sigma)^{F(m, t)^{-1}} \quad (6)$$

Consequently, Bob will believe that the forged SDVS  $(t'', \sigma'')$  for  $m''$  is generated by Alice.

### 4. An Efficient and Provably Secure Improvement

To withstand above universal forgery attacks, we can adopt a cryptographic hash function,  $h: G_2 \rightarrow G_2$ , to rewrite Equation 3 as:

$$\sigma = h(e(T, Q_B)) \quad (7)$$

Then the corresponding Equation 4 would become:

$$\sigma = h(t \cdot e(Q_A, S_B)^{F(m, t)}) \quad (8)$$

Hence, the universal forgery attack cannot work any longer in the improved mechanism, as any malicious adversary is not able to derive the shared secret  $e(Q_A, S_B)$ . The underlining security notion of Kang *et al.*'s scheme and our improvement is based on the

Bilinear Diffie-Hellman Problem (BDHP) stated below:

- *Bilinear Diffie-Hellman Problem*: The BDHP is, given an instance  $(P, X, Y, Z) \in G_1^4$  where  $P$  is a generator,  $X = xP, Y = yP$  and  $Z = zP$  for some  $x, y, z \in Z_q$ , to compute  $e(P, P)^{xyz} \in G_2$ .
- *Bilinear Diffie-Hellman (BDH) Assumption*: For every probabilistic polynomial-time algorithm  $D$ , every positive polynomial  $Q(\cdot)$  and all sufficiently large  $k$ , the algorithm  $D$  can solve the BDHP with the advantage at most  $1/Q(k)$ , i. e.,  $\Pr[D(P, xP, yP, zP) = e(P, P)^{xyz}; x, y, z \leftarrow Z_q, (P, xP, yP, zP) \leftarrow G_1^4] \leq 1/Q(k)$ .

The probability is taken over the uniformly and independently chosen instance and over the random choices of  $\mathcal{D}$ .

- *Definition 1*: The  $(t, \varepsilon)$ -BDH assumption holds if there is no polynomial-time adversary that can solve the BDHP in time at most  $t$  and with the advantage  $\varepsilon$ .

By applying the similar proof techniques of Kang *et al.*'s scheme, we can also, formally prove the security of our improved mechanism in the random oracle model as follows:

- *Theorem 1*: The improved SDVS scheme is  $(t, q_H, q_F, q_h, q_{Extract}, q_S, q_V, \varepsilon)$ -secure against EF-CMA in the random oracle model if there is no probabilistic polynomial-time adversary that can  $(t', \varepsilon')$ -break the BDHP, where:  $\varepsilon' \geq 2(\varepsilon - 2^{-|G_2|}) (q_S^2 - q_S)^{-1}, t' \approx t + t_\lambda(2q_S + q_V)$ .

Here,  $t_\lambda$  is the time for performing one bilinear pairing operation.

- *Proof*: Suppose that a probabilistic polynomial-time adversary  $D$  can  $(t, q_H, q_F, q_h, q_{Extract}, q_S, q_V, \varepsilon)$ -break the improved SDVS scheme with non-negligible advantage  $\varepsilon$  under adaptive chosen message attacks after running at most  $t$  steps and making at most  $q_H H, q_F F, q_h h, q_{Extract}$  KeyExtract,  $q_S$  Sign and  $q_V$  Verify oracle queries. Then we can construct another algorithm  $C$  that can  $(t', \varepsilon')$ -break the BDHP by taking  $D$  as a subroutine. The objective of  $C$  is to obtain  $e(P, P)^{abc}$  by taking  $(P, aP, bP, cP)$  as inputs. For all the queries of  $(H, F, KeyExtract, Sign, Verify)$ ,  $C$  responds as those defined in Kang *et al.*'s scheme, i. e.,  $P_{TA} = bP, Q_A = aP, Q_B = cP$ , etc., When  $D$  queries an  $h$  oracle of  $h(z)$ ,  $C$  first checks the  $h\_list$  for a matched entry. Otherwise,  $C$  chooses  $v \in_R G_2$ , adds the entry  $(z, v)$  to the  $h\_list$ , and returns  $v$  as a result.

Finally,  $D$  outputs a valid forgery  $(t^*, \sigma^*)$  for  $m^*$  with respect to the signer's identity  $ID_i$  and the designated verifier's identity  $ID_j$ .  $C$  first searches the  $h\_list$  for a matched entry  $(z, \sigma)$  where  $\sigma = \sigma^*$  and then outputs the value  $(t^{*-1}z)^{F(m^*, t^*)^{-1}} = e(P, P)^{abc}$  as the answer to the BDHP. The probability that  $D$  guesses the correct

random value without asking  $h(z^*)$  oracle is not greater than  $2^{-|G_2|}$ . Besides, the probability that  $(i, j) = \{(A, B) \text{ or } (B, A)\}$  is  $2(q_s(q_s - 1))^{-1} = 2(q_s^2 - q_s)^{-1}$ . Therefore, we can express the probability that  $C$  solves the BDHP as  $\varepsilon' \geq 2(\varepsilon - 2^{-|G_2|})(q_s^2 - q_s)^{-1}$ . The computational steps required for  $C$  are  $t' \approx t + t_i(2q_s + q_v)$ .

## 5. Conclusions

Although, Kang *et al.*'s identity-based SDVS scheme has the advantages of lower computational costs and shorter signature length. They also, formally proved the security of their scheme in the random oracle model. Nevertheless, we demonstrated that their scheme still cannot resist universal forgery attacks for arbitrarily chosen messages. Additionally, we gave an efficient and provably secure improvement by adopting a cryptographic hash function to eliminate such a security weakness. It is evident that the improved mechanism also, preserves the computational and communicational merits of Kang *et al.*'s scheme.

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